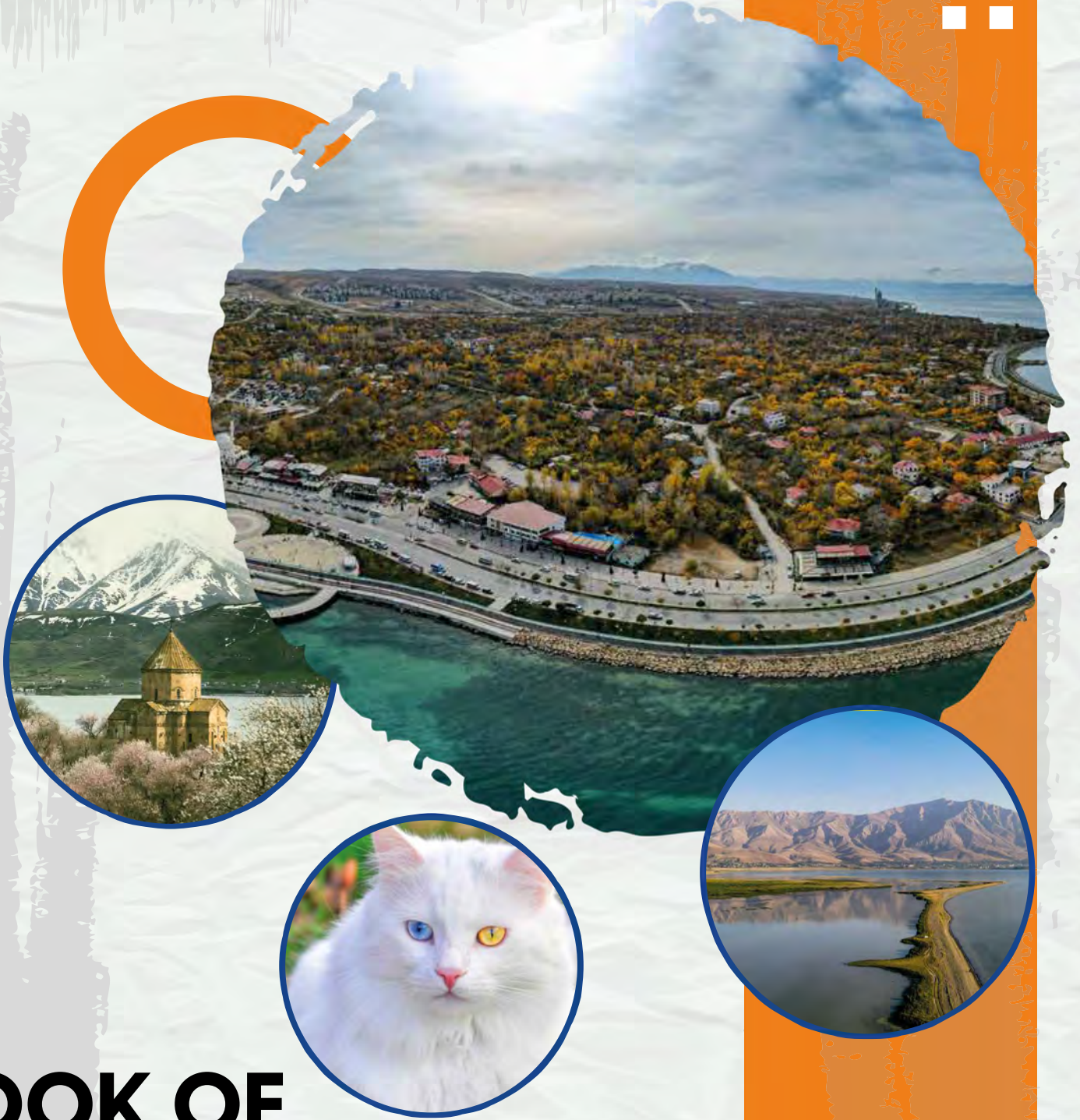




ICPAM-VAN 2022

4th International Conference on
Pure and Applied Mathematics



**BOOK OF
ABSTRACTS**

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Van Yüzüncü Yıl University

4th International Conference on Pure and
Applied Mathematics (ICPAM - VAN 2022)

BOOK OF ABSTRACTS

June 22-23, 2022
Van, Turkey

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Van Yüzüncü Yıl University, Van, TURKEY,
June 22-23, 2022

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Welcome

Dear Participants,

Welcome to 4th International Conference on Pure and Applied Mathematics (ICPAM - VAN 2022), Van, Turkey. The conference is organized at Van Yüzüncü Yıl University from June 22nd to June 23th. Since safety and health of our guests, staff and community are of utmost importance to us, at this time, we have made the difficult decision to postpone all travels, as well as participations in ICPAM - VAN 2022 that involve large gatherings. Due to the Covid-19 concerns, this year the fourth ICPAM - VAN 2022 becomes a virtual conference, that is, it turns into an interactive seminar conducted over the internet (webinar) as the third one in 2020 while the first and the second ones were organized as face to face meetings in 2015 and 2018, respectively. The reason for our 3-year long break is to be a host of another high involvement conference.

The purpose of the virtual conference is to provide a platform where researchers in the field of pure and applied mathematics can present their researches, exchange new ideas, discuss challenging issues, foster future collaborations and interact with each other.

With 12 invited speaker from 11 different countries, 6 parallel sessions (totally 29 sessions), 132 presentations and more than 130 participants from 16 countries, Algeria, Australia, Austria, Egypt, Germany, India, Iran, Italy, Oman, Portugal, Republic of Korea, Romania, Spain, Turkey, United Arab Emirates, USA, as well as people from 50 different universities from Turkey, ICPAM - VAN 2022 will provide a stimulating opportunity for a global interchange of ideas on recent advances in mathematics.

I would like to express my deepest gratitude to Prof. Dr. Hamdullah ŞEVLİ, President of Van Yüzüncü Yıl University, and Prof. Dr. Esvet AKBAŞ, Dean of Faculty of Science of Van Yüzüncü Yıl University, for their encouragement and support in all stages of this conference.

I am grateful to all the members of the Scientific and Organizing Committees, the referees and the authors for producing such a high standard conference.

I would also like to thank to the sponsors, Van Yüzüncü Yıl University and Abdullah Gül University for their generous support.

The last but not the least, I would like to give our heartfelt thanks to participants for their understanding, patience and contributions which make our conference more successful. Since the conference is almost entirely from the registration support of participants, I am grateful for their financial support as well. Thank you all again for the unbelievable amount of support and understanding you have shared during this period. It means more than words can express. We, the members of organizing committee and I, were honored and happy to organize this online meeting which provides online interactions between mathematicians who can not meet, talk and discuss face to face and offers an online collaboration and online social networks.

I wish all of you to stay healthy and safe. I look forward to the day we welcome you all back to Van, city of the sun.

Professor Cemil Tunç
Organizing Committee Chair of ICPAM - VAN 2022

Contents

Welcome	4
Scientific programme in details	10
June 22, 2022	10
Scientific programme table	15
June 23, 2022	16
Scientific programme table	23
Abstracts of invited speakers	24
Ayman BADAWI - N-zero-divisor graph of a commutative semigroup	25
Adil BAGIROV - Nonsmooth DC optimization: Methods and applications	27
Martin BOHNER - Hyers–Ulam and Hyers–Ulam–Rassias stability of first-order linear and nonlinear dynamic equations	28
Ahmed Mohamed EL-SAYED - Stochastic fractional calculus operators: Basic concepts and applications	29
Soon-Mo JUNG - Generalized linear span and its applications	30
Alireza KHALILI GOLMANKHANEH - Fractal calculus needs to include fractal measure	31
Salvador LÓPEZ-ALFONSO, Manuel LÓPEZ-PELLICER, Santiago MOLL-LÓPEZ - On Nikodým, Grothendieck, Vitali-Hahn-Saks and Valdivia measure theorems	32
Alessandro PAOLUCCI, Cristina PIGNOTTI - Exponential decay for semilinear damped wave equations with delay feedback	33
Choonkil PARK - Almost hom-Poisson algebras and homomorphisms and derivations on hom-Poisson algebras	34
Sandra PINELAS - Oscillation criteria for second order neutral delay differential equations	35
Hans-Peter SCHRÖCKER - Nested bases for rational PH-curves	36
Vitalii SLYNKO - Construction of a Lyapunov function for 1-D linear hyperbolic 2×2 systems	37
Abstracts of participants' talks	39
Garvita AGRAWAL, Manjeet Singh TEETH, K. N. RAJESWARI - Properties of generalized Fibonacci and Lucas polynomials	40
N. Rosa AIT-AMRANE, Hacene BELBACHIR, Elif TAN - A study on r -Fibonacci and r -Lucas hybrid polynomials	41
Ilker AKKUS, Betül ERDOĞAN - Cholesky and LU algorithms of Lucas type matrices	42
Ilker AKKUS, Safak YENIAYDIN - Spectral properties of some matrices with binomial coefficients	43
Mehmet Serif ALDEMİR, Suleyman EDİZ, Murat CANSAN - On the stratified domination number of generalized Petersen graphs	44
Imran ALI, Javid IQBAL - Second order partial differential evolutionary equations driven by variational-like inequalities	45
Nagehan ALSOY-AKGÜN - The use of dual reciprocity boundary element method for solving the coupled nonlinear Sine-Gordon equations	46
Ilhame AMIRALI, Hülya ACAR - Stability analysis for high-order Volterra delay integro-differential equation	47
Zineb ARAB, Cemil TUNÇ - On some stochastic time-fractional integral equations	48
İrem ARIK, Cemil TUNÇ - On the existence of positive periodic solutions of nonlinear neutral differential equations	49
Berna ARSLAN - Stability of Jordan k -derivations on Γ -Banach algebras	50
Okan ARSLAN, Ahmet GENÇ - On the source of semi-primeness of Γ -rings	51

Merve ARTIRAN, Zülfükar SAYGI - Roman type domination parameters of Lucas cubes	52
Enes ATA - M-extension of Lauricella hypergeometric functions and their integral representations	53
Muzaffer ATEŞ - Boundedness of solutions to a nonlinear differential equation of fourth order	54
Muzaffer ATEŞ, Rodi AYID ALI - Lyapunov stability analysis of electrical power systems	55
Harun BİÇER - Mahgoub transform method in stability research	56
Murat BODUR, Prashantkumar PATEL - Integral modification of Lupaş type operators	57
Lydia BOUCHAL, Karima MEBARKI - Multiple fixed point results for sum of operators and application	58
Zeynep CAN - A note on some spaces induced by convex polyhedra and their isometry groups	59
Sibel CANSU, Erol YILMAZ, Uğur USTAOĞLU - Divided domains satisfy the weakly radical formula	60
Akram CHEHRAZI - An overview of twistor theory	61
Idris CIFTCI - QSPR investigation of leap Randić index of octanes	62
Idris CIFTCI, Suleyman EDIZ - On k-total distance degrees and related indices of graphs	63
Mustafa COSKUN - Eigen gaps to decide on graph convolutional network architecture	64
Nimet COSKUN - Spectral singularities of the Sturm-Liouville equations with a non-standard density function	65
Gökçen ÇEKİÇ - Bifurcation scheme of multi-layer flows on various wave velocities	66
Ayfer ÇETE, Yasin YAZLIK - On a solvable system of difference equations with constant coefficients	67
Pakize ÇETİN, Okan KUZU - Examination of preservice mathematics teachers' written expression skills for geometric objects: Student diaries	68
Şahap ÇETİN, Yalçın YILMAZ, Coşkun YAKAR - Quasilinearization method for a generalized initial-value problem on the time scale	69
Zübeyir ÇINKIR - Explicit and recursive formulas for Kirchhoff index of graphs via integer sequences such as generalized Fibonacci numbers	70
Özge DALMANOĞLU - Approximation properties of a generalization of Szász-Beta operators	71
Cahit DEDE - New infinite classes of Laplacian borderenergetic graphs	72
Ekin DELIKTAS-OZDEMİR, Ayse PEKER DOBIE - Some comparisons between the effects of planar and corrugated interfaces of a layer sandwiched between two half spaces on SH wave propagation	73
Yılmaz Mehmet DEMIRCI - Crumbling modules as injectivity domains	74
Nihal DEMİR, Hafize GÜMÜŞ - Lacunary statistical convergence of multiset sequences	75
Ceyda DEMİREL - The nabla z-transform	76
İsmail Hakkı DENİZLER - Artinian analogues of some Noetherian modules over complete semi-local commutative Noetherian rings	77
Yavuz DİNÇ, Erhan PİŞKİN, Cemil TUNÇ - Upper bounds for the blow up time for the Kirchhoff-type equation	78
Sibel DOGRU AKGÖL - New oscillation/ nonoscillation criteria for impulsive differential equations with discontinuous solutions	79
Hülya DURU, Serkan İLTER, Aygül BİLGİN - A first countable Hausdorff topology induced by a quasi-uniformity	80
Ramazan EKMEKÇİ - Almost continuity and almost compactness in graded ditopological texture spaces	81
Elif ERÇELİK, Mustafa NADAR - Nonnegative bias reduction methods for density estimation using scaled inverse chi-squared kernel estimator	82
Sultan ERDUR, Cemil TUNÇ - Stability, boundedness and existence of periodic solutions of some fourth order non-linear differential equations with multiple delays	83
Sare Buse ERSU - On stability of linear difference systems	84
Figen ERYILMAZ - Cofinitely ss -lifting modules	85

Beyaz Basak ESKISEHIRLI - An operator-theoretic setting of singly-generated invariant subspaces in the polydisc	86
Cansu EVCİN - Optimal control on the MHD mixed convection flow with transverse magnetic field	87
Boutheina FELLAHI, Bachir MERIKHI - A logarithmic barrier interior point method for linear programming	88
Şehmus FINDIK - On automorphisms of symmetric polynomials of noncommutative algebras	89
Kerem GEZER, Mine MENEKŞE YILMAZ - The Voronovskaja type theorem for a class of Kantorovich type operators associated with the Charlier polynomials I	90
Melek GÖZEN - New qualitative criteria to certain vector differential equations of second order	91
Ghania GUETTAI, Mourad RAHMANI - A new class of $m - (p, q)$ -Bernoulli polynomials .	92
Baransel GUNES, Musa CAKIR - A fully discrete scheme for singularly perturbed semilinear integro-differential equations with two integral boundary conditions	93
Betül GÜDÜK İBRAHİMOĞLU, Mustafa Taylan ŞENGÜL - On the analysis of a model with Holling type-III functional response consisting of super predator, intermediate predator and prey	94
Kübra GÜL - On the Perrin hybrid quaternions	95
Rabia GÜLERYÜZ, Burak UYAR - Introduction to Grey models	96
Hafize GÜMÜŞ - A generalized form of ρ -statistical convergence of interval numbers . . .	97
Çisem GÜNEŞ AKTAŞ - Arithmetical reduction of classification problems of singular K3-surfaces	98
Mehmet GÜRDAL, Hamdullah BAŞARAN - Advanced refinements of Berezin number inequalities	99
Ali HAMİDOĞLU - On designing discrete-time games from continuous-time models	100
Ilham HASSI, Mohamed Amine BOUTICHE - The detour index of graph products	101
Karim HEDAYATIAN, Mohammad NAMEGOSHAYFARD - On commutant hypercyclicity and commutant transitivity of some operators	102
Amel HIOUAL, Adel OUANNAS, Taki Eddine OUSSAEIF, Zineb LAOUAR - On fractional variable-order neural networks based on the Caputo derivative	103
İlhan İÇEN, Abdullah Fatih ÖZCAN - Every gobal action defines a local equivalence relation	104
Seda İĞRET ARAZ - Applications of the piecewise derivative to real-world problems . . .	105
Serkan İLTER, Hülya DURU, Seyit KOCA, Havva Nur ÖZTÜRK - A C^1 approximation on differential inclusions	106
Merve KARA, Ömer AKTAŞ - On solutions of three-dimensional system of difference equations	107
Neriman KARTAL - Stability and bifurcation analysis of a COVID-19 mathematical model on Erdős Rényi network	108
Gülistan KAYA GÖK - Different bounds for degree Kirchhoff index	109
Zeynep KAYAR, Billur KAYMAKÇALAN - Novel delta and nabla Pachpatte type inequalities via convexity	110
Billur KAYMAKÇALAN, Zeynep KAYAR - Diamond-alpha Pachpatte type dynamic inequalities	111
Nurten KILIÇ - On oscillation of solutions of advanced differential equations	112
Gonca KIZILASLAN - Factorization of a statistical matrix	113
Mustafa Kutay KUTLU, Engin MERMUT - Roth's theorem on arithmetic progressions . .	114
Ömer KÜSMÜŞ - A note on nilpotent units in commutative group rings of direct product groups	115
Zineb LAOUAR, Nouria ARAR, Amel HIOUAL - On the numerical solution of the fractional integro-differential equation using shifted Chebyshev polynomials of the third kind	116

Murat LUZUM, İsmail Hakkı DENİZLER - On semicommutative rings in term of nilpotent elements	117
Jasbir Singh MANHAS - Boundedness of sum of generalized weighted composition operators between weighted spaces of analytic functions	118
Kasım MANSIZ, Cemil TUNÇ- On the behavior of solutions of Riemann-Liouville fractional nonlinear equation with multiple variable delays	119
Sanda MICULA- A superconvergent collocation method for two-dimensional Hammerstein integral equations	120
Oktay Sh. MUKHTAROV, Merve YUCEL, Kadriye AYDEMİR - Approximate solution of the transmission problem via transform method	121
Ali MUSALİ, Çağrı SAĞLAM, Mustafa Kerem YÜKSEL - Hopf bifurcations in a model of epidemic awareness	122
Burcu NİŞANCI TÜRKMEN, Yılmaz Mehmet DEMİRCİ - Injective modules are ss-lifting in R -mod	123
Zozan OKTAN, Cemil TUNÇ- A new result on the asymptotic stability of a stochastic delay differential equation of the second order	124
Abdullah Fatih ÖZCAN, İlhan İÇEN- Topological group-groupoids	125
Tufan ÖZDİN- The minus partial order on endomorphism rings	126
Seyed Zeynal PASHAEI, Necat GORENTAS- Topological group and small loop transfer space	127
Erhan PİŞKİN, Ruken AKSOY- Existence and nonexistence for the higher-order logarithmic Klein-Gordon equation	128
Erhan PİŞKİN, Erkan SANCAR- Existence and decay for the logarithmic Lamé system	129
Asghar RAHIMI, Bayaz DARABY- Multipliers of frames and woven frames in Hilbert spaces	130
Mehdi RASHIDI-KOUCHI - On controlled frame multipliers in Hilbert spaces	131
Mehdi RASHIDI-KOUCHI, Maryam MOHAMMADREZAEI, Akbar NAZARI - Woven continuous g -fusion frames in Hilbert spaces	132
Kelthoum Lina REDOUANE, Nouria ARAR - Cubic b-spline collocation technique for time-dependent problems with small perturbation parameter	133
Mehmet Gıyas SAKAR - A new method for variable order fractional Volterra-Fredholm integro-differential equations	134
Mine SARIGÜL, Ercan TUNÇ - New oscillation criteria for odd-order neutral differential equations with distributed deviating arguments	135
Nil ŞAHİN - Conjectures of Rossi and Sally on the monotonicity of the Hilbert functions	136
Nilay ŞAHİN BAYRAM - Some Korovkin theorems for linear operators via power series method	137
Pınar ŞAŞMAZ, Murad ÖZKOÇ, Santanu ACHARJEE - On ωe^* -continuous functions	138
Merve ŞENGÜN, Cemil TUNÇ - On the Hyers-Ulam stability of nonlinear Volterra integro-differential equation with the multiple constant delays	139
Sinem ŞİMŞEK - A block conjugate gradient method for linear positive definite quaternion matrix equations	140
Şerife TEKİN, Aynur KESKİN KAYMAKCI - On Maki's Λ -sets via strong β - I -open set	141
Hayri TOPAL - Weighted composition operators on weighted spaces of holomorphic functions on Banach spaces	142
Ali Hakan TOR - Benchmarking for nonsmooth convex optimization methods	143
Osman TUNÇ - A note on qualitative behaviors of solutions of integro-differential equations	144
Cemil TUNÇ, Osman TUNÇ - Enhanced new qualitative criteria to delay fractional integro-differential equations with Caputo derivative	145
Ergül TÜRKMEN, Yılmaz Mehmet DEMİRCİ - Wsa-coinjective modules	146
Cennet TÜRKÖĞLU, Abdulcabbar SÖNMEZ - Some new sequence spaces derived by the composition of weighted mean and quadruple band matrix	147

Gopal YADAV, Rajesh Kumar SHARMA, Gendlal PRAJAPATI - Complex valued controlled fuzzy metric spaces and some common fixed point results	148
Tülay YAĞMUR - On bicomplex numbers with coefficients from higher order Fibonacci numbers	149
Ferit YALAZ, Aynur KESKİN KAYMAKCI - New continuity types via local closure function	150
Ruhi YALÇIN, Handan KÖSE - A new characterization of reflexive rings and their applications	151
Reha YAPALI, Erdal KORKMAZ - On statistical convergence for triple sequences on L -fuzzy normed space	152
Cumali YAŞAR, Ahmet Buğra YAŞAR- Quantum computer-resistant digital signature generation using complex numbers	153
Ramazan YAZGAN - Analysis of global exponential stability and pseudo almost periodic solution of a class of chaotic neural networks on time scales	154
Nurullah YILMAZ, Hatice OGUT - An exact penalty function approach for inequality constrained optimization problems based on a new smoothing technique	155
Melis Berçin YILMAZ, Zülfükar SAYGI - Roman type domination numbers of Fibonacci cubes	156
Ceyda YILMAZ LUZUM, Şenay BAYDAŞ, Bülent KARAKAŞ - Bezier curves belonging to some surfaces	157
Abdullah YİĞİT - Improved new results on the asymptotic admissibility of nonlinear singular systems with multiple delays	158
Halil İbrahim YOLDAŞ - On the geometry of Ricci solitons	159

Scientific programme in details

June 22, 2022

Room 1 (Invited speakers) Chair: Cemil TUNÇ

09:00-09:15 **Opening Ceremony**

09:15-10:00 **Adil BAGIROV**

Nonsmooth DC optimization: Methods and applications

10:00-10:45 **Choonkil PARK**

Almost hom-Poisson algebras and homomorphisms and derivations on hom-Poisson algebras

10:45-11:00 Coffee Break

11:00-11:45 **Ayman BADAWI**

N-zero-divisor graph of a commutative semigroup

11:45-12:30 **Ahmed Mohamed EL-SAYED**

Stochastic fractional calculus operators: Basic concepts and applications

12:30-13:40 Lunch Break

Room 1 Chair: Ramazan YAZGAN

13:40-14:00 **Zeynep KAYAR**, Billur KAYMAKÇALAN

Novel delta and nabla Pachpatte type inequalities via convexity

14:00-14:20 **Billur KAYMAKÇALAN**, Zeynep KAYAR

Diamond-alpha Pachpatte type dynamic inequalities

14:20-14:40 **Ceyda DEMİREL**

The nabla z-transform

14:40-15:00 **Sare Buse ERSU**

On stability of linear difference systems

15:00-15:10 Coffee Break

Room 1 Chair: Cemil TUNÇ

15:10-15:30 **Merve ŞENGÜN**, Cemil TUNÇ

On the Hyers-Ulam stability of nonlinear Volterra integro-differential equation with the multiple constant delays

15:30-15:50 **Melek GÖZEN**

New qualitative criteria to certain vector differential equations of second order

15:50-16:10 **Ramazan YAZGAN**

Analysis of global exponential stability and pseudo almost periodic solution of a class of chaotic neural networks on time scales

16:10-16:30 **İrem ARIK**, Cemil TUNÇ

On the existence of positive periodic solutions of nonlinear neutral differential equations

16:30-16:45 Coffee Break

Room 1 (Invited speakers) Chair: Cemil TUNÇ

16:45-17:30 **Hans-Peter SCHRÖCKER**

Nested bases for rational PH-curves

17:30-18:15 **Sandra PINELAS**

Oscillation criteria for second order neutral delay differential equations

18:15-19:00 **Martin BOHNER**

Hyers-Ulam and Hyers-Ulam-Rassias stability of first-order linear and nonlinear dynamic equations

June 22, 2022

Room 2 Chair: Osman TUNÇ

13:40-14:00 **Cemil TUNÇ**, Osman TUNÇ

Enhanced new qualitative criteria to delay fractional integro-differential equations with Caputo derivative

14:00-14:20 **Kasım MANSIZ**, Cemil TUNÇ

On the behavior of solutions of Riemann-Liouville fractional nonlinear equation with multiple variable delays

14:20-14:40 **Zozan OKTAN**, Cemil TUNÇ

A new result on the asymptotic stability of a stochastic delay differential equation of the second order

14:40-15:00 **Harun BİÇER**

Mahgoub transform method in stability research

15:00-15:10 Coffee Break

Room 2 Chair: İsmail Hakkı DENİZLER

15:10-15:30 **Ferit YALAZ**, Aynur KESKİN KAYMAKCI

New continuity types via local closure function

15:30-15:50 **Şerife TEKİN**, Aynur KESKİN KAYMAKCI

On Maki's Λ -sets via strong β - I -open set

15:50-16:10 **Pınar ŞAŞMAZ**, Murad ÖZKOÇ, Santanu ACHARJEE

On ωe^* -continuous functions

16:10-16:30 Hülya DURU, Serkan İLTER, **Aygül BİLGİN**

A first countable Hausdorff topology induced by a quasi-uniformity

16:30-16:45 Coffee Break

June 22, 2022**Room 3** Chair: Nagehan AKGÜN

- 13:40-14:00 **Sinem ŞİMŞEK**
A block conjugate gradient method for linear positive definite quaternion matrix equations
- 14:00-14:20 **Ali Hakan TOR**
Benchmarking for nonsmooth convex optimization methods
- 14:20-14:40 **Baransel GÜNEŞ**, Musa ÇAKIR
A fully discrete scheme for singularly perturbed semilinear integro-differential equations with two integral boundary conditions
- 14:40-15:00 **Seda İĞRET ARAZ**
Applications of the piecewise derivative to real-world problems
- 15:00-15:10 Coffee Break

Room 3 Chair: Ali Hakan TOR

- 15:10-15:30 **Cansu EVCİN**
Optimal control on the MHD mixed convection flow with transverse magnetic field
- 15:30-15:50 **Nagehan AKGÜN**
The use of dual reciprocity boundary element method for solving the coupled nonlinear Sine-Gordon equations
- 15:50-16:10 **Kelthoum Lina REDOUANE**, Nouria ARAR
Cubic b-spline collocation technique for time-dependent problems with small perturbation parameter
- 16:10-16:30 Ali MUSALİ, Çağrı SAĞLAM, **Mustafa Kerem YÜKSEL**
Hopf bifurcations in a model of epidemic awareness
- 16:30-16:45 Coffee Break

June 22, 2022

Room 4 Chair: Murat LUZUM

13:40-14:00 **Zübeyir ÇINKIR**

Explicit and recursive formulas for Kirchhoff index of graphs via integer sequences such as generalized Fibonacci numbers

14:00-14:20 **Mehmet Şerif ALDEMİR**, Süleyman EDİZ, Murat CANSAN

On the stratified domination number of generalized Petersen graphs

14:20-14:40 **Gülistan KAYA GÖK**

Different bounds for degree Kirchhoff index

14:40-15:00 **Ilham HASSI**, Mohamed Amine BOUTICHE

The detour index of graph products

15:00-15:10 Coffee Break

Room 4 Chair: Zeynep KAYAR

15:10-15:30 **Nilay ŞAHİN BAYRAM**

Some Korovkin theorems for linear operators via Power Series Method

15:30-15:50 **Cennet TÜRKÖĞLU**, Abdulcabbar SÖNMEZ

Some new sequence spaces derived by the composition of weighted mean and quadruple band matrix

15:50-16:10 **Nihal DEMİR**, Hafize GÜMÜŞ

Lacunary statistical convergence of multiset sequences

16:10-16:30 **Hafize GÜMÜŞ**

A generalized form of ρ -statistical convergence of interval numbers

16:30-16:45 Coffee Break

June 22, 2022**Room 5** Chair: Yılmaz Mehmet DEMİRCİ13:40-14:00 **Yılmaz Mehmet DEMİRCİ**

Crumbling modules as injectivity domains

14:00-14:20 **Ergül TÜRKMEN**, Yılmaz Mehmet DEMİRCİ

Wsa-coinjective modules

14:20-14:40 **Ruhi YALÇIN**, Handan KÖSE

A new characterization of reflexive rings and their applications

14:40-15:00 **Tufan ÖZDİN**

The minus partial order on endomorphism rings

15:00-15:10 Coffee Break

Room 5 Chair: Çisem GÜNEŞ AKTAŞ15:10-15:30 **Murat LUZUM**, İsmail Hakkı DENİZLER

On semicommutative rings in term of nilpotent elements

15:30-15:50 **Ömer KÜSMÜŞ**

A note on nilpotent units in commutative group rings of direct product groups

15:50-16:10 **Figen ERYILMAZ**Cofinitely ss -lifting modules16:10-16:30 **Burcu NIŞANCI TÜRKMEN**, Yılmaz Mehmet DEMİRCİInjective modules are ss -lifting in R -mod

16:30-16:45 Coffee Break

Room 6 Chair: İsmail Hakkı DENİZLER13:40-14:00 Mehmet GÜRDAL, **Hamdullah BAŞARAN**

Advanced renements of Berezin number inequalities

14:00-14:20 **Beyaz Başak ESKİŞEHİRLİ**

An operator-theoretic setting of singly-generated invariant subspaces in the polydisc

14:20-14:40 **Hayri TOPAL**

Weighted composition operators on weighted spaces of holomorphic functions on Banach spaces

14:40-15:00 **Mehdi RASHIDI-KOUCHI**

On controlled frame multipliers in Hilbert spaces

15:00-15:10 Coffee Break

Room 6 Chair: Osman TUNÇ15:10-15:30 **Lydia BOUCHAL**, Karima MEBARKI

Multiple fixed point results for sum of operators and application

15:30-15:50 **Jasbir Singh MANHAS**

Boundedness of sum of generalized weighted composition operators between weighted spaces of analytic functions

15:50-16:10 **Gopal YADAV**, Rajesh Kumar SHARMA, Gendlal PRAJAPATI

Complex valued controlled fuzzy metric spaces and some common fixed point results

16:10-16:30 Karim HEDAYATIAN, **Mohammad NAMEGOSHAYFARD**

On commutant hypercyclicity and commutant transitivity of some operators

16:30-16:45 Coffee Break

June 22, 2022 Wednesday						
Time						
Chair	CEMİL TUNÇ					
09:00-09:15	Opening Ceremony					
09:15-10:00	Invited Speaker					
10:00-10:45	Invited Speaker					
10:45-11:00	Coffee Break					
Chair	CEMİL TUNÇ					
11:00-11:45	Invited Speaker					
11:45-12:30	Invited Speaker					
12:30-13:40	Lunch Break					
Chair	Room 1	Room 2	Room 3	Room 4	Room 5	Room 6
13:40-14:00	RAMAZAN YAZGAN	OSMAN TUNÇ	NAGEHAN AKGÜN	MURAT LUZUM	YILMAZ DEMİRCİ	İSMAIL DENİZLER
14:00-14:20	BİLLUR KAYMAKÇALAN	CEMİL TUNÇ	SİNEM ŞİMŞEK	ZÜBEYİR ÇINKİR	YILMAZ DEMİRCİ	HAMDULLAH BAŞARAN
14:20-14:40	CEYDA DEMİREL	KASIM MANSIZ	ALİ HAKAN TOR	M. ŞERİF ALDEMİR	ERGÜL TÜRKMEN	BEYAZ BAŞAK ESKİŞEHİRLİ
14:40-15:00	SARE BUSE ERSU	ZOZAN OKTAN	BARANSEL GÜNEŞ	GÜLİSTAN KAYA GÖK	RUHİ YALÇIN	HAYRİ TOPAL
15:00-15:10		HARUN BİÇER	SEDA İCRET ARAZ	İLHAM HASSI	TUFAN ÖZDİN	MEHDI RASHIDI-KOUCHI
Chair	Room 1	Room 2	Room 3	Room 4	Room 5	Room 6
15:10-15:30	CEMİL TUNÇ	İSMAIL DENİZLER	ALİ HAKAN TOR	ZEYNEP KAYAR	ÇİSEM AKTAŞ	OSMAN TUNÇ
15:30-15:50	MERVE ŞENGÜN	FERİT YALAZ	CANSU EVCİN	NİLAY ŞAHİN BAYRAM	MURAT LUZUM	LYDIA BOUCHAL
15:50-16:10	MELEK GÖZEN	ŞERİFE TEKİN	NAGEHAN AKGÜN	CENNET TÜRKÖĞLU	ÖMER KÜSMÜŞ	JASBİR SINGH MANHAS
16:10-16:30	RAMAZAN YAZGAN	PINAR ŞAŞMAZ	KELTHOUM LINA RE-DOUANE	NİHAL DEMİR	FİĞEN ERYILMAZ	GOPAL YADAV
16:30-16:45	İREM ARIK	AYGÜL BİLGİN	MUSTAFA KEREM YÜKSEL	HAFİZE GÜMÜŞ	BURCU NİŞANCI TÜRK-MEN	MOHAMMAD NAMEGOSHAYFARD
Chair	CEMİL TUNÇ					
16:45-17:30	Invited Speaker					
17:30-18:15	Invited Speaker					
18:15-19:00	Invited Speaker					

June 23, 2022

Room 1 (Invited speakers) Chair: Cemil TUNÇ

08:45-09:30 **Soon-Mo JUNG**

Generalized linear span and its applications

09:30-10:15 **Alireza KHALILI GOLMANKHANEH**

Fractal calculus needs to include fractal measure

10:15-10:30 Coffee Break

10:30-11:15 Alessandro PAOLUCCI, **Cristina PIGNOTTI**

Exponential decay for semilinear damped wave equations with delay feedback

11:15-12:00 **Vitalii I. SLYNKO**

Construction of a Lyapunov function for 1-D linear hyperbolic 2×2 systems

12:00-12:45 Salvador LÓPEZ-ALFONSO, **Manuel LÓPEZ-PELLICER**, Santiago MOLL-LÓPEZ

On Nikodým, Grothendieck, Vitali-Hahn-Saks and Valdivia measure theorems

12:45-14:00 Lunch Break

Room 1 Chair: Osman TUNÇ

14:00-14:20 Muzaffer ATEŞ, **Rodi AYID ALI**

Lyapunov stability analysis of electrical power systems

14:20-14:40 **Amel HIOUAL**, Adel OUANNAS, Taki Eddine OUSSAEIF, Zineb LAOUAR

On fractional variable-order neural networks based on the Caputo derivative

14:40-15:00 **Betül GÜDÜK İBRAHİMOĞLU**, Mustafa Taylan ŞENGÜL

On the analysis of a model with Holling type-III functional response consisting of super predator, intermediate predator and prey

15:00-15:20 **Muzaffer ATEŞ**

Boundedness of solutions to a nonlinear differential equation of fourth order

15:20-15:30 Coffee Break

Room 1 Chair: Ramazan YAZGAN

15:30-15:50 **Osman TUNÇ**

A note on qualitative behaviors of solutions of integro-differential equations

15:50-16:10 Serkan İLTER, Hülya DURU, **Seyit KOCA**, Havva Nur ÖZTÜRK

A C^1 approximation on differential inclusions

16:10-16:30 **Şahap ÇETİN**, Yalçın YILMAZ, Coşkun YAKAR

Quasilinearization method for a generalized initial-value problem on the time scale

16:30-16:50 **Abdullah YİĞİT**

Improved new results on the asymptotic admissibility of nonlinear singular systems with multiple delays

16:50-17:10 **Zineb LAOUAR**, Nouria ARAR, Amel HIOUAL

On the numerical solution of the fractional integro-differential equation using shifted Chebyshev polynomials of the third kind

17:10-17:20 Coffee Break

Room 1 Chair: Osman TUNÇ

- 17:20-17:40 Oktay Sh. MUKHTAROV, **Merve YUCEL**, Kadriye AYDEMİR
Approximate solution of the transmission problem via transform method
- 17:40-18:00 **Sibel DOĞRU AKGÖL**
New oscillation/ nonoscillation criteria for impulsive differential equations with discontinuous solutions
- 18:00-18:20 **Mine SARIGÜL**, Ercan TUNÇ
New oscillation criteria for odd-order neutral differential equations with distributed deviating arguments
- 18:20-18:40 İlham AMIRALI, **Hülya ACAR**
Stability analysis for high-order Volterra delay integro-differential equation
- 18:40-19:00 **Sultan ERDUR**, Cemil TUNÇ
Stability, boundedness and existence of periodic solutions of some fourth order nonlinear differential equations with multiple delays
- 19:00-19:15 Closing Ceremony

June 23, 2022**Room 2** Chair: Hayri TOPAL14:00-14:20 **Murat BODUR**, Prashantkumar PATEL

Integral modification of Lupaş type operators

14:20-14:40 **Kerem GEZER**, Mine MENEKŞE YILMAZ

The Voronovskaja type theorem for a class of Kantorovich type operators associated with the Charlier polynomials I

14:40-15:00 **Özge DALMANOĞLU**

Approximation properties of a generalization of Szász-Beta operators

15:00-15:20 **Sanda MICULA**

A superconvergent collocation method for two-dimensional Hammerstein integral equations

15:20-15:30 Coffee Break

Room 2 Chair: Hayri TOPAL15:30-15:50 Erhan PİŞKİN, **Ruken AKSOY**

Existence and nonexistence for the higher-order logarithmic Klein-Gordon equation

15:50-16:10 Erhan PİŞKİN, **Erkan SANCAR**

Existence and decay for the logarithmic Lamé system

16:10-16:30 **Yavuz DİNÇ**, Erhan PİŞKİN, Cemil TUNÇ

Upper bounds for the blow up time for the Kirchhoff-type equation

16:30-16:50 **Zineb ARAB**, Cemil TUNÇ

On some stochastic time-fractional integral equations

16:50-17:10 **Gökçen ÇEKİÇ**

Bifurcation scheme of multi-layer flows on various wave velocities

17:10-17:20 Coffee Break

Room 2 Chair: Zeynep KAYAR17:20-17:40 **Okan ARSLAN**, Ahmet GENÇOn the source of semi-primeness of Γ -rings17:40-18:00 **Cahit DEDE**

New infinite classes of Laplacian borderenergetic graphs

18:00-18:20 **Melis BERÇİN YILMAZ**, Zülfükar SAYGI

Roman type domination numbers of Fibonacci cubes

18:20-18:40 İdris ÇİFTÇİ, **Süleyman EDİZ**

On k-total distance degrees and related indices of graphs

18:40-19:00 **İdris ÇİFTÇİ**

QSPR investigation of leap Randić index of octanes

June 23, 2022

Room 3 Chair: Ömer KÜSMÜŞ

- 14:00-14:20 **Ekin DELİKTAŞ-ÖZDEMİR**, Ayse PEKER DOBIE
Some comparisons between the effects of planar and corrugated interfaces of a layer sandwiched between two half spaces on SH wave propagation
- 14:20-14:40 **Imran ALI**, Javid IQBAL
Second order partial differential evolutionary equations driven by variational-like inequalities
- 14:40-15:00 **Rabia GÜLERYÜZ**, Burak UYAR
Introduction to Grey models
- 15:00-15:20
- 15:20-15:30 Coffee Break

Room 3 Chair: Çisem GÜNEŞ AKTAŞ

- 15:30-15:50 **Mustafa Kutay KUTLU**, Engin MERMUT
Roth's theorem on arithmetic progressions
- 15:50-16:10 **Tülay YAĞMUR**
On bicomplex numbers with coefficients from higher order Fibonacci numbers
- 16:10-16:30 **Merve ARTIRAN**, Zülfükar SAYGI
Roman type domination parameters of Lucas cubes
- 16:30-16:50 **Garwita AGRAWAL**, Manjeet Singh TEETH, K. N. RAJESWARI
Properties of generalized Fibonacci and Lucas polynomials
- 16:50-17:10 **Reha YAPALI**, Erdal KORKMAZ
On statistical convergence for triple sequences on L-fuzzy normed space
- 17:10-17:20 Coffee Break

Room 3 Chair: Ömer KÜSMÜŞ

- 17:20-17:40 **Kübra GÜL**
On the Perrin hybrid quaternions
- 17:40-18:00 Ilker AKKUS, **Betül ERDOĞAN**
Cholesky and LU algorithms of Lucas type matrices
- 18:00-18:20 Ilker AKKUS, **Şafak YENİAYDIN**
Spectral properties of some matrices with binomial coefficients
- 18:20-18:40 **Ghania GUETTAI**, Mourad RAHMANI
A new class of m -(p,q)-Bernoulli polynomials
- 18:40-19:00 **N. Rosa AIT-AMRANE**, Hacene BELBACHIR, Elif TAN
A study on r -Fibonacci and r -Lucas hybrid polynomials

June 23, 2022

Room 4 Chair: Murat LUZUM

14:00-14:20

14:20-14:40

14:40-15:00 **Şehmus FINDIK**

On automorphisms of symmetric polynomials of noncommutative algebras

15:00-15:20

15:20-15:30 Coffee Break

Room 4 Chair: Murat LUZUM

15:30-15:50 Merve KARA, **Ömer AKTAŞ**

On solutions of three-dimensional system of difference equations

15:50-16:10 **Ayfer ÇETE**, Yasin YAZLIK

On a solvable system of difference equations with constant coefficients

16:10-16:30 **Neriman KARTAL**

Stability and bifurcation analysis of a COVID-19 mathematical model on Erdős Rényi network

16:30-16:50 **Nimet COŞKUN**

Spectral singularities of the Sturm-Liouville equations with a nonstandard density function

16:50-17:10 **Berna ARSLAN**

Stability of Jordan k -derivations on Γ -Banach algebras

17:10-17:20 Coffee Break

Room 4 Chair: Nagehan AKGÜN

17:20-17:40 **Çisem GÜNEŞ AKTAŞ**

Arithmetical reduction of classification problems of singular K3- surfaces

17:40-18:00 **Gonca KIZILASLAN**

Factorization of a statistical matrix

18:00-18:20 **Nil ŞAHİN**

Conjectures of Rossi and Sally on the monotonicity of the Hilbert functions

18:20-18:40 **İsmail Hakkı DENİZLER**

Artinian analogues of some Noetherian modules over complete semi-local commutative Noetherian rings

18:40-19:00 **Sibel CANSU**, Erol YILMAZ, Uğur USTAOĞLU

Divided domains satisfy the weakly radical formula

June 23, 2022

Room 5 Chair: Nagehan AKGÜN

14:00-14:20 **Halil İbrahim YOLDAŞ**

On the geometry of Ricci solitons

14:20-14:40 **Zeynep CAN**

A note on some spaces induced by convex polyhedra and their isometry groups

14:40-15:00 **Akram CHEHRAZI**

An overview of twistor theory

15:00-15:20 **Ceyda YILMAZ LUZUM, Şenay BAYDŞ, Bülent KARAKAŞ**

Bezier curves belonging to some surfaces

15:20-15:30 Coffee Break

Room 5 Chair: Ali Hakan TOR

15:30-15:50 Mehdi RASHIDI-KOUCHI, **Maryam MOHAMMADREZAEI**, Akbar NAZARI
Woven continuous g-fusion frames in Hilbert spaces

15:50-16:10 **Asghar RAHIMI**, Bayaz DARABY

Multipliers of frames and woven frames in Hilbert spaces

16:10-16:30 Cumali YAŞAR, **Ahmet Buğra YAŞAR**

Quantum computer-resistant digital signature generation using complex numbers

16:30-16:50 **Mustafa COŞKUN**

Eigen gaps to decide on graph convolutional network architecture

16:50-17:10

17:10-17:20 Coffee Break

Room 5 Chair: Ramazan YAZGAN

17:20-17:40 **Pakize ÇETİN**, Okan KUZU

Examination of preservice mathematics teachers' written expression skills for geometric objects: Student diaries

17:40-18:00 **Nurten KILIÇ**

On oscillation of solutions of advanced differential equations

18:00-18:20 **Mehmet Gıyas SAKAR**

A new method for variable order fractional Volterra-Fredholm integro-differential equations

18:20-18:40 **Seyd Zeynal PASHAEI**, Necat GORENTAS

Topological group and small loop transfer space

June 23, 2022

Room 6 Chair: Çisem GÜNEŞ AKTAŞ

- 14:00-14:20 **İlhan İÇEN**, Abdullah Fatih ÖZCAN
Every global action defines a local equivalence relation
- 14:20-14:40 **Abdullah Fatih ÖZCAN**, İlhan İÇEN
Topological group-groupoids
- 14:40-15:00 **Ramazan EKMEKÇİ**
Almost continuity and almost compactness in graded ditopological texture spaces
- 15:00-15:20 **Enes ATA**
M-extension of Lauricella hypergeometric functions and their integral representations
- 15:20-15:30 Coffee Break

Room 6 Chair: Ömer KÜSMÜŞ

- 15:30-15:50 **Elif ERÇELİK**, Mustafa NADAR
Nonnegative bias reduction methods for density estimation using scaled inverse chi-squared kernel estimator
- 15:50-16:10 **Nurullah YILMAZ**, Hatice OGUT
An exact penalty function approach for inequality constrained optimization problems based on a new smoothing technique
- 16:10-16:30 **Ali HAMİDOĞLU**
On designing discrete-time games from continuous-time models
- 16:30-16:50 **Boutheina FELLAHI**, Bachir MERIKHI
A logarithmic barrier interior point method for linear programming
- 16:50-17:10
- 17:10-17:20 Coffee Break

Abstracts of invited speakers

N-zero-divisor graph of a commutative semigroup

AYMAN BADAWI

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Let S be a (multiplicative) commutative semigroup with 0, $Z(S)$ the set of zero-divisors of S , and n a positive integer. The classical *zero-divisor graph* of S is the (simple) graph $\Gamma(S)$ with vertices $Z(S)^* = Z(S) \setminus \{0\}$, and distinct vertices x and y are adjacent if and only if $xy = 0$. In this talk, we introduce and study the *n-zero-divisor graph* of S as the (simple) graph $\Gamma_n(S)$ with vertices $Z_n(S)^* = \{x^n \mid x \in Z(S)\} \setminus \{0\}$, and distinct vertices x and y are adjacent if and only if $xy = 0$. Thus each $\Gamma_n(S)$ is an induced subgraph of $\Gamma(S) = \Gamma_1(S)$. We pay particular attention to $\text{diam}(\Gamma_n(S))$, $\text{gr}(\Gamma_n(S))$, and the case when S is a commutative ring with $1 \neq 0$.

MSC 2010: 13A70, 05C25, 05C99, 13A15, 13B99

Keywords: Annihilator graph, commutative ring with identity, commutative semigroup with zero, congruence-based zero-divisor graph, extended zero-divisor graph

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Nonsmooth DC optimization: Methods and applications

ADIL BAGIROV

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In this talk we consider unconstrained and constrained optimization problems where the objective and/or constraint functions are represented as a difference of two convex (DC) functions. We discuss two different approaches to design methods of nonsmooth DC optimization: an approach based on the extension of bundle methods of nonsmooth optimization and an approach based on the DCA (difference of convex algorithm). We present numerical results and discuss various applications of DC optimization in machine learning.

MSC 2010: 90C26, 65K05, 68T05

Keywords: Nonsmooth optimization, nonconvex optimization, DC functions, Machine learning

Hyers–Ulam and Hyers–Ulam–Rassias stability of first-order linear and nonlinear dynamic equations

MARTIN BOHNER

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We present several new sufficient conditions for Hyers–Ulam and Hyers–Ulam–Rassias stability of first-order linear and nonlinear dynamic equations for functions defined on a time scale with values in a Banach space.

MSC 2010: 34N05, 39A30, 39A12

Keywords: Hyers–Ulam stability, linear dynamic equation, nonlinear dynamic equation

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Stochastic fractional calculus operators: Basic concepts and applications

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In this talk, we give the basic concepts and definitions of the fractional-order integral and fractional order derivatives of the second order stochastic processes. The main properties of these operators will be given. Some problems of stochastic and random differential equations will be studied. The combination between the fractional order derivative and stochastic Ito-differential and integral will be studied in some coupled systems of nonlocal stochastic and random differential equations.

MSC 2010: 34A12, 34A30, 34D20

Keywords: Stochastic processes, stochastic fractional-order derivatives, stochastic fractional-order integrals

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Generalized linear span and its applications

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The notions of first-order and second-order generalized linear spans and index set are defined. Moreover, their properties are investigated and applied to the studies of extension of isometries. We develop the theory of extending the domain of local isometries to the generalized linear spans, where we call an isometry defined in a subset of a Hilbert space a local isometry.

MSC 2010: 46B04, 46C99

Keywords: isometry, extension of isometry, generalized linear span

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Fractal calculus needs to include fractal measure

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This paper presents fractal calculus which is based on the measure of fractals and ordinary calculus. We consider functions that are defined on the Cantor sets, then we show that ordinary calculus can not be used for that function because of two reasons. First, because the length measure used in the ordinary calculus is not suitable for fractals, for example, in the case of the Cantor set, the length is zero. Secondly, the Cantor set is totally discontinuous therefore the functions are not integrable. But we explain how fractal calculus is solving these two problems. More, Fractal Laplace equations and fractal random variables are presented [1, 2, 3, 4, 5, 6, 7, 8].

MSC 2010: 28A80, 28A20

Keywords: Fractal calculus, Cantor sets, fractal derivative, fractal Laplace equations, fractal random variables

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On Nikodým, Grothendieck, Vitali-Hahn-Saks and Valdivia measure theorems

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Let \mathcal{A} be an algebra of subsets of a set Ω and let $ba(\mathcal{A})$ the Banach space of real or complex bounded finitely additive measures defined on \mathcal{A} endowed with the variation norm, which is equivalent to the supremum norm.

A subset \mathcal{B} of \mathcal{A} has property (N) if each \mathcal{B} -pointwise bounded sequence $(\mu_n, n \in \mathbb{N})$ of $ba(\mathcal{A})$ is norm bounded in $ba(\mathcal{A})$, i.e., the sequence $(\mu_n, n \in \mathbb{N})$ is uniformly bounded in \mathcal{A} . \mathcal{B} has property (G) $[(VHS)]$ if for each bounded sequence [if for each sequence] $(\mu_n, n \in \mathbb{N})$ of $ba(\mathcal{A})$ the \mathcal{B} -pointwise convergence of $(\mu_n, n \in \mathbb{N})$ to $\mu \in ba(\mathcal{A})$ implies its weak convergence, i.e., $\lim_{n \rightarrow \infty} \varphi(\mu_n) = \varphi(\mu)$ for each linear continuous form φ defined in $ba(\mathcal{A})$.

\mathcal{B} has property (sN) $[(sG)$ or $(sVHS)]$ if every increasing covering $\{\mathcal{B}_n : n \in \mathbb{N}\}$ of \mathcal{B} contains a set \mathcal{B}_p with property (N) $[(G)$ or $(VHS)]$, and \mathcal{B} has property (wN) $[(wG)$ or $(wVHS)]$ if for every increasing web $\{\mathcal{B}_{n_1 n_2 \dots n_m} : n_i \in \mathbb{N}, 1 \leq i \leq m, m \in \mathbb{N}\}$ of \mathcal{B} there exists sequence $(p_n, n \in \mathbb{N})$ of natural numbers such that for each natural number m the set $\mathcal{B}_{p_1 p_2 \dots p_m}$ property (N) $[(G)$ or $(VHS)]$ for every $m \in \mathbb{N}$.

The classical theorems of Nikodým-Grothendieck, Valdivia, Grothendieck and Vitali-Hahn-Saks say, respectively, that every σ -algebra \mathcal{A} has properties (N) , (sN) , (G) and (VHS) . The main objective of this conference is to expose our recent results that every σ -algebra \mathcal{A} has properties (wN) , (wG) and $(wVHS)$, improving the mentioned four theorems. Moreover applications of these new properties are considered as well as the characterization that a subset \mathcal{B} of an algebra \mathcal{A} has property $(wWHS)$ if and only if \mathcal{B} has property (wN) and \mathcal{A} has property (G) and several open questions.

MSC 2010: 28A60, 46G10

Keywords: Algebra and σ -algebra of subsets, bounded finitely additive scalar measure, Nikodým property, Grothendieck property, Vitali-Hahn-Saks property

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Exponential decay for semilinear damped wave equations with delay feedback

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We analyze a class of semilinear damped wave-type equations with a delay feedback with time-variable coefficient. By combining semigroup arguments, careful energy estimates, and an iterative approach we are able to prove, under suitable assumptions, a well-posedness result and an exponential decay estimate for solutions corresponding to small initial data.

MSC 2010: 93D15, 35L90, 5B35

Keywords: Semilinear wave equations, time delays, feedback stabilization, exponential stability

Almost hom-Poisson algebras and homomorphisms and derivations on hom-Poisson algebras

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We construct hom-Poisson algebra from almost hom-Poisson algebra by using the Hyers-Ulam stability method and prove the Hyers-Ulam stability of homomorphisms and derivations on hom-Poisson algebras by using the direct method and the fixed point method. We moreover investigate the properties of hom-Poisson algebras.

MSC 2010: 17A15, 17B63, 17B99

Keywords: Hom-Poisson algebra, almost hom-Poisson algebra

Oscillation criteria for second order neutral delay differential equations

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The authors present some sufficient conditions for the oscillation of second order neutral delay differential equation

$$(a(t)(z'(t))^\beta)' + q(t)x^\gamma(\sigma(t)) = 0, \quad t \geq t_0 > 0,$$

where $z(t) = x(t) + p(t)x(\tau(t))$. The neutral differential equations find wide range of applications in certain high-tech fields, such as control theory, mechanical engineering, physics, population dynamics, economics, ... From these discussions, we can see that the investigation of oscillatory and asymptotic behavior of solutions of second order neutral delay differential equations is of great importance in both theory and applications.

MSC 2010: 34C10, 34K11

Keywords: Neutral differential equation, second order, oscillation

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Nested bases for rational PH-curves

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This is joint work with Bahar Kalkan, Daniel Scharler and Zbyněk Šír.

A curve with rational parametric equations is called “Pythagorean hodograph” or “PH” if its normalized derivative vector is rational as well. PH-curves are useful in situations where not only a geometric point locus but also its time-dependent trajectory need to be modeled. Examples include motion control, animation, or path planning.

Polynomial PH-curves have been studied extensively for more than thirty years. There is an elegant direct approach for their computation by integration of certain vectorial quaternion polynomials. Rational PH-curves are usually computed as envelopes of their osculating planes. While this yields a closed formula for all rational PH-curves, it is difficult to control their degree as unexpected cancellations often occur. The reason for these cancellations is hidden in some convoluted polynomial algebra and is not yet fully understood.

We present an alternative method for computing rational and polynomial PH-curves. Given the curve’s tangent indicatrix and its denominator polynomial, it requires solving a modestly sized system of linear equations. Rather surprisingly, it turns out that polynomial PH-curves are generic in this context and rational PH-curves occur only in special cases. With our approach, PH-curves of bounded degree naturally form an infinite sequence of nested vector spaces for which bases can be computed. This not only gives new insight into the structure of polynomial and rational PH-curves. It also suggested a very clear standardized representation via finite Laurent series and provides computational advantages for applications.

MSC 2010: 65D17

Keywords: Pythagorean-hodograph curves, rational and polynomial PH-curves, tangent indicatrix

Construction of a Lyapunov function for 1-D linear hyperbolic 2×2 systems

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Consider a linear not strictly hyperbolic 2×2 system

$$\begin{aligned}\partial_t x_1(z, t) &= a(z)\partial_z x_1(z, t) + b_{11}(z)x_1(z, t) + b_{12}(z)x_2(z, t), \\ \partial_t x_2(z, t) &= a(z)\partial_z x_2(z, t) + b_{21}(z)x_1(z, t) + b_{22}(z)x_2(z, t)\end{aligned}\quad (1)$$

with the boundary conditions

$$\begin{pmatrix} x_1(\ell, t) \\ x_2(\ell, t) \end{pmatrix} = \Lambda \begin{pmatrix} x_1(0, t) \\ x_2(0, t) \end{pmatrix}.\quad (2)$$

For the system (1)—(2), we consider the Cauchy problem with initial conditions $(x_1(z, 0), x_2(z, 0)) = (x_{10}(z), x_{20}(z))$. We assume, that $a \in C^1([0, \ell], \mathbb{R})$, $b_{ij} \in C^1([0, \ell], \mathbb{R})$, $i, j = 1, 2$, $i \neq j$, $a(z) > 0$ for all $z \in [0, \ell]$, and $\Lambda = (\lambda_{ij})_{i,j=1,2}$ is a constant matrix.

A Lyapunov function of the following structure

$$V(x_1, x_2) = \int_0^\ell (q_1(z)x_1^2(z) + q_2(z)x_2^2(z)) dz, \quad i, j = 1, 2, \quad i \neq j,$$

where $q_i \in C^1([0, \ell], \mathbb{R}_{>0})$ is called the diagonal Lyapunov function [1]. We establish necessary conditions for the existence of the diagonal Lyapunov function.

Theorem 1 *Let for the linear system (1)—(2) there exists the diagonal Lyapunov function, then the following inequalities hold*

$$\begin{aligned}|\lambda_{11}| &< \exp\left(-\int_0^\ell \frac{b_{11}(z)}{a(z)} dz\right), \quad |\lambda_{22}| < \exp\left(-\int_0^\ell \frac{b_{22}(z)}{a(z)} dz\right), \\ |\lambda_{12}||\lambda_{21}| &< \exp\left(\int_0^\ell \frac{b_{11}(z) + b_{22}(z) - 2\sqrt{(b_{12}(z)b_{21}(z))_+}}{a(z)} dz\right), \\ \lambda_{11}\lambda_{12}\lambda_{21}\lambda_{22} &< \frac{1}{2} \exp\left(2\int_0^\ell \frac{b_{11}(z) + b_{22}(z) - 2\sqrt{(b_{12}(z)b_{21}(z))_+}}{a(z)} dz\right).\end{aligned}$$

Here $(f(z))_+ = \begin{cases} f(z), & f(z) \geq 0, \\ 0, & f(z) < 0. \end{cases}$

A non-diagonal Lyapunov function of the following form

$$V(x) = \int_0^\ell e^{\mu z} x^T(z) P(z) x(z) dz, \quad P(z) = \frac{1}{a(z)} \Omega_0^z P_0 (\Omega_0^z)^T$$

is proposed, where $\mu > 0$, Ω_0^z is the solution of the following linear matrix differential equation

$$\frac{d\Omega_0^z}{dz} = \frac{1}{a(z)} B^T(z) \Omega_0^z, \quad \Omega_0^0 = \text{Id}, \quad B(z) = (b_{ij}(z))_{i,j=1,2},$$

P_0 is a positive definite matrix. This Lyapunov function leads to the following conditions for the asymptotic stability of the system (1)—(2). We denote $r_\sigma(\cdot)$ is a spectral radius of a corresponding matrix.

Theorem 2 *Let $r_\sigma((\Omega_0^\ell)^T \Lambda) < 1$, then the linear system (1)—(2) is exponentially stable.*

Since the explicit form of matrices Ω_0^z is usually unknown, we proposed the non-diagonal Lyapunov function with the kernel

$$P(z) = \frac{1}{a(z)} W^T(z) P_0 W(z),$$

P_0 is a positive definite matrix, $W(z) = \exp U(z)$, where $U(z)$ is the partial sum of the Magnus series [2]

$$\begin{aligned} U(z) = & \int_0^z C(\tau) d\tau - \frac{1}{2} \int_0^z \left[C(\tau), \int_0^\tau C(\sigma) d\sigma \right] d\tau \\ & + \frac{1}{4} \int_0^z \left[C(\tau), \int_0^\tau \left[C(\sigma), \int_0^\sigma C(\rho) d\rho \right] d\sigma \right] d\tau \\ & + \frac{1}{12} \int_0^z \left[\left[C(\tau), \int_0^\tau C(\sigma) d\sigma \right], \int_0^\tau C(\sigma) d\sigma \right] d\tau. \end{aligned}$$

Based on this Lyapunov function sufficient conditions of the asymptotic stability of the system (1)—(2) are obtained. We have compared various exponential stability conditions obtained using the diagonal and non-diagonal Lyapunov functions.

MSC 2010: 35L04, 37L15, 37L45

Keywords: Linear 1-D hyperbolic systems, exponential stability, Magnus series

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Abstracts of participants' talks

Properties of generalized Fibonacci and Lucas polynomials

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The renowned Fibonacci and Lucas polynomials possess various astounding properties and identities. The Fibonacci polynomial has been generalized in many ways by conserving the recurrence relation and others by preserving the initial conditions. In this paper we have defined generalized Fibonacci and Lucas polynomial and with the help of generating function and Binet's formula, we proved famous identities for the same in our settings.

MSC 2010: 11B37, 11B39, 11B50

Keywords: Fibonacci polynomial, Lucas polynomial, generalized Fibonacci polynomial, generalized Lucas polynomial, generating function

A study on r -Fibonacci and r -Lucas hybrid polynomials

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The hybrid numbers was introduced by Ozdemir [4] as a generalization of complex, dual and hyperbolic numbers.

The set of hybrid numbers is defined as

$$\mathbb{K} = \{a + bi + c\varepsilon + dh \mid i^2 = -1, \varepsilon^2 = 0, h^2 = 1, a, b, c, d \in \mathbb{R}\}, \quad (1)$$

with i, ε, h are the complexe, the dual and the hyperbolic units, respectively where $ih = -hi = \varepsilon + i$.

In this work [1], we introduce the r -Fibonacci and r -Lucas hybrid polynomials. We investigate some basic properties of these hybrid polynomials. Also, we establish their matrix representation. We derive generalized Cassini's identity and generalized Honsberger formula for r -Fibonacci hybrid polynomials by using their matrix representation.

MSC 2010: 11B39, 05A15, 11R52

Keywords: r -Fibonacci polynomial, r -Lucas polynomial, hybrid number

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Cholesky and LU algorithms of Lucas type matrices

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Horadam investigated the generalized Fibonacci sequence $W_n(a, b; p, q)$, where a, b, p, q are integers $W_0 = a, W_1 = b$ and

$$W_{n+2} = pW_{n+1} + qW_n, \quad n \geq 2.$$

For $a = 0, b = 1$ and $q = 1$, we use the recurrence $W_n(a, b; p, q)$ in the form

$$U_{n+2} = pU_{n+1} + U_n, \quad n \geq 2.$$

For $a = 2, b = 1$ and $q = 1$, we use the form $W_n(a, b; p, q)$ in the form

$$V_{n+2} = pV_{n+1} + V_n, \quad n \geq 2.$$

The main purpose of this study is to use a 2-by-2 matrix determined by the *Cholesky* and *LU* decomposition algorithms to construct a collection of identities including U_n and V_n . Some of these identities, extend the results obtained elsewhere articles.

Some well-known examples of the use of the 2-by-2 matrix, covering summation of some finite series including U_n and V_n , are showed. A way for calculating some infinite series is then offered which is based on the use of a closed form expression for certain matrix functions.

MSC 2010: 11B39, 15A23

Keywords: Factorization of matrices; generalized Fibonacci and Lucas numbers; recurrences

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Spectral properties of some matrices with binomial coefficients

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Characteristic polynomial and trace properties of the right-justified binomial matrix and its inverse had been introduced by L. Carlitz in 1965 [1]. When the right-justified binomial matrix and its inverse are added or subtracted, another binomial coefficients matrices are obtained. This property is our the mainly motivation on making a research on this study. The second one is when studied on eigenvalues, it's seen that, they can be identified in terms of Lucas and Fibonacci numbers. Beside that, eigenvalues of these matrices make some periodic sequences in modulo 3, modulo 5 and modulo 7 similar to case in [2] and this explains constructions of characteristic polynomials in modulo 3, modulo 5 and modulo 7. Since similar properties can be seen in characteristic polynomial of the right-justified binomial matrix, this polynomial is studied on a paper [3] published by P.Stanica and R.Peele. Finally, owing to eigenvalues can be expressed in terms of Lucas and Fibonacci numbers, these numbers are seen in trace and determinant values too.

MSC 2010: 11B65, 15A15, 15A18

Keywords: Binomial coefficients, eigenvalues and eigenvectors, determinants

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On the stratified domination number of generalized Petersen graphs

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Stratified graphs have important role in VLSI design in electronic engineering science. The vertex set of any graph is called two stratified graph when its vertex set partitioned into two disjunction sets. Various domination parameters have been calculated for generalized Petersen graphs so far. In this study we firstly investigated two stratified domination number for the generalized Petersen graphs $GP(n, 1)$ and $GP(n, 2)$. We presented our results in two closed formulas.

MSC 2010: 05C07, 05C69, 05C90

Keywords: Domination, stratified domination, generalized Petersen graphs

Second order partial differential evolutionary equations driven by variational-like inequalities

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In this paper, we introduce and study a system by mixing an evolutionary second order partial differential equation and a mixed variational-like inequality in Banach spaces, which is called second order evolutionary partial differential variational-like inequality ((SOEPDVLI), in short). The solutions for (SOEPDVLI) in the mild sense is given by using the theory of strongly continuous cosine family of bounded linear operator. We have proved that the solution set of mixed variational-like inequalities involved in (SOEPDVLI) is nonempty, bounded, closed and convex. We have proved that the mapping $\mathcal{U} : [0, T] \times Y \times Y \rightarrow \Pi_{bv}(X)$ is upper semicontinuous and measurable. Finally, an existence result for (SOEPDVLI) is obtained by using fixed point theorem for condensing set-valued operators and theory of measures of non-compactness.

MSC 2010: 49J40, 34G25, 49K40

Keywords: Second order evolution equation, Variational-like inequality, Cosine family, Measure of non-compactness, Mild solution

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The use of dual reciprocity boundary element method for solving the coupled nonlinear Sine-Gordon equations

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In this study, the dual reciprocity boundary element method (DRBEM) is used for the solution of two-dimensional time dependent coupled sine-Gordon equations. The form of the modified Helmholtz equations are obtained by using central difference approximations for both first and second order time derivatives and by inserting the finite difference approximations into the governing equation. The fundamental solution of modified Helmholtz equation is employed in the integral equation formulation. The inhomogeneous terms of the equation cause a domain integral in the boundary integral equation. The DRBEM provides to transform the domain integral into the boundary integral by approximating with thin plate spline the inhomogeneous term of the equation ($r^2 \ln r$). The main aim of the current study is to show that DRBEM is also suitable for solving the system of coupled nonlinear sine-Gordon equations. Several test problems are employed and results of numerical experiments are presented and also are compared with analytical solutions. Presented numerical results are observed to be in good agreement with other numerical results given in [1] and [2].

MSC 2010: 65M38, 65N38, 35J05

Keywords: Coupled sine-Gordon equation, DRBEM, FDM, modified Helmholtz equation

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Stability analysis for high-order Volterra delay integro-differential equation

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High-order linear Volterra delay integro-differential equations are examined in the present paper. Proposed approach, which will be provided for solving high-order linear Volterra delay integro-differential equations, expresses certain key elements of determining the equations' stability bounds and exact solutions. Furthermore, stability inequalities can be generated for each order of derivative using the proposed method.

Consider the following high-order Volterra delay integro-differential equation:

$$u^{(n)}(t) + a(t)u^{(n-1)}(t) + b(t)u(t-r) + \int_{t-r}^t K(t,s)u(s)ds = f(t) \quad (1)$$

on $(0, T]$ with the interval conditions:

$$u(t) = \varphi(t), \quad u^{(i)}(0) = B_i, \quad i = 1, 2, \dots, n-1 \quad \text{and} \quad t \in I_0, \quad (2)$$

where $I = (0, T] = \cup_{p=1}^m I_p$, $I_p = \{t : r_{p-1} < t \leq r_p\}$, $1 \leq p \leq m$ and $r_s = sr$, for $0 \leq s \leq m$, $\bar{I} = [0, T]$, $I_0 = [-r, 0]$, $a(t) \geq 0$, $f(t)$, $a(t)$ ($t \in \bar{I}$), $\varphi(t)$ ($t \in I_0$) and $K(t, s)$ ($(t, s) \in \bar{I} \times \bar{I}$) are provided sufficiently smooth functions that satisfy specific regularity conditions to be described, r is a constant delay. Moreover, we will assume that $a, b, f \in C(\bar{I})$, $\varphi \in C^2(I_0)$ and $\frac{\partial^2 K}{\partial s^2} \in C(\bar{I}^2)$ ($s = 0, 1, 2$).

MSC 2010: 34K28, 45J05, 65L05

Keywords: Volterra integro-differential equation, high-order delay integro-differential equation, stability inequality

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On some stochastic time-fractional integral equations

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In the current work, we deal with a class of stochastic time-fractional integral equations in Hilbert space by studying their wellposedness and regularity. Precisely, we use the celebrity fixed point theorem to prove the wellposedness of the problem by imposing the global Lipschitz and the linear growth conditions. Further, we prove the spatial and the temporal regularity by imposing only a regularity condition on the initial value. An important example is considered in order to confirm and to support the validity of our theoretical results.

MSC 2010: 35B65, 35R11, 35R60, 60H15, 65M75

Keywords: Integral equations, Riemann-Liouville integral operator, cylindrical Wiener process, fixed point theorem, spatial regularity, temporal regularity

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On the existence of positive periodic solutions of nonlinear neutral differential equations

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In this paper, we discuss the existence of positive periodic solutions of a nonlinear neutral differential equation by using Krasnoselskii's fixed point theory. A new result, which include sufficient conditions, is proved on the existence of periodic solutions of the consider equation. By this work, we generalized some results in the literature. The new result of this paper makes contribution to topic of the existence of positive periodic solution. We give also an example for illustrations.

MSC 2010: 34K20, 34K06, 34K40.

Keywords: Neutral differential equation, fixed point theorem, positive periodic solutions

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Stability of Jordan k -derivations on Γ -Banach algebras

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The stability of functional equations was first introduced by S.M. Ulam [4] in 1940. Since then several results concerning the Hyers-Ulam-Rassias stability of various functional equations with more general domains and ranges have been extensively investigated by many mathematicians. The concept of a Γ -ring was introduced by N. Nobusawa [3] and generalized by W.E. Barnes [1]. D.K. Bhattacharya and A.K. Maity [2] gave the definition of a Γ -Banach algebra in 1989. Γ -Banach algebras are generalization of both the concepts of Banach algebras and Γ -rings. Let V be a Γ -Banach algebra over the complex field \mathbb{C} and let $D : V \rightarrow V$ and $k : \Gamma \rightarrow \Gamma$ be two linear mappings. If $D(a\alpha a) = D(a)\alpha a + ak(\alpha)a + a\alpha D(a)$ for all $a \in V$, $\alpha \in \Gamma$, then D is called a Jordan k -derivation of V .

In this talk, we will investigate the Hyers-Ulam-Rassias stability and superstability of Jordan homomorphisms and Jordan k -derivations on Γ -Banach algebras by using fixed point methods for the following Jensen-type functional equation $\mu f\left(\frac{x+y}{2}\right) + \mu f\left(\frac{x-y}{2}\right) = f(\mu x)$ where μ is a complex number such that $|\mu| = 1$.

MSC 2010: 39B52, 39B82, 16W25, 16Y99

Keywords: Hyers-Ulam-Rassias stability, superstability, Γ -Banach algebra, k -derivation, Jordan k -derivation

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On the source of semi-primeness of Γ -rings

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The notion of a Γ -ring was introduced by N. Nobusawa [1] in 1964, and then W.E. Barnes [2] generalized Nobusawa's definition by weakening the conditions and developed the more general Γ -ring in which all the classical rings are contained in this Γ -ring. Since then the theory of Γ -rings have been extensively investigated by a number of mathematicians to develop many basic characterizations of Γ -rings.

Semi-prime ideals and prime ideals are important for determining properties of the structure of Γ -rings. Many mathematicians make use of these concepts while working in this field. Therefore, the notion of the source of semi-primeness is a candidate to give important results for these structures.

The source of semi-primeness for usual rings was first defined by N. Aydın, Ç. Demir and D. K. Camcı in [3]. In this talk, we introduce the notion of the source of semi-primeness of Γ -rings as the subset $S_M = \{a \in M : a\Gamma M\Gamma a = (0)\}$ of M and we give the definition of $|S_M|$ -reduced Γ -ring M . Then, we investigate some basic results of these notions for Γ -rings.

MSC 2010: 05C25, 13A15, 13M05

Keywords: Gamma ring, semi-primeness, reduced gamma ring

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Roman type domination parameters of Lucas cubes

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The n -dimensional hypercube graph denoted by Q_n is one of the basic models for interconnection networks. The vertices of Q_n are represented by all binary strings of length n and two vertices are adjacent if and only if they differ in exactly one position. By removing all the vertices containing two consecutive 1's and the vertices that start and end with 1 from the vertex set of Q_n , n -dimensional Lucas cubes are obtained for $n \geq 2$ [1].

The domination problem is one of the hard problem in graph theory Let G be a graph and the D be a subset of the vertex set $V(G)$ of G . D is a called a dominating set if every vertex from $V(G)$ either belongs to D or adjacent to some vertex from D . The domination number of G is the minimum cardinality of a dominating set of G . The domination number can be used to reduce costs and increase speed in areas such as transportation network and food service. For a general graph it is hard to determine its domination number.

A Roman dominating function of a graph G is a labeling $f : V(G) \rightarrow \{0, 1, 2\}$ such that every vertex with label 0 has a neighbor with label 2 [2]. The Roman domination number $\gamma_R(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over all Roman dominating functions. Later, different types of Roman dominating sets were formed by specialization. In this work, we examine three different Roman domination type parameters and obtain their exact values for Lucas cubes of up to 10 dimensions using Integer Linear Programming.

MSC 2010: 11B39, 05C69, 68R10

Keywords: Lucas cube, Domination number, Roman domination number

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M-extension of Lauricella hypergeometric functions and their integral representations

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In this paper, we introduce new extensions of Lauricella hypergeometric function $F_A^{(r)}$, Lauricella hypergeometric function $F_B^{(r)}$, Lauricella hypergeometric function $F_C^{(r)}$, and Lauricella hypergeometric function $F_D^{(r)}$, by using the modified beta function, which given with the generalized M-series in its kernel. Furthermore, we obtained various integral formulas for the newly defined extended Lauricella hypergeometric function $F_A^{(r)}$, extended Lauricella hypergeometric function $F_B^{(r)}$, extended Lauricella hypergeometric function $F_C^{(r)}$, and extended Lauricella hypergeometric function $F_D^{(r)}$.

MSC 2010: 33B15, 33C15, 33C65

Keywords: Beta function, confluent hypergeometric function, generalized M-series, Lauricella's hypergeometric functions

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Boundedness of solutions to a nonlinear differential equation of fourth order

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We consider the boundedness of the solution to a certain nonlinear differential equation of fourth-order. We prove that the solution and their derivatives up to order five are bounded by using the Cauchy formula for the solution of the nonhomogeneous differential equation. In this investigation, we will give two auxiliary lemmas and one main result.

MSC 2010: 34D23; 34D20; 34C23

Keywords: Fourth- order differential equation, boundedness of the solutions, Cauchy formula

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Lyapunov stability analysis of electrical power systems

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In this paper, we consider the properties of the power system stability of multi-machine system by Lyapunov direct method which is one of the most powerful tools for determining the behavior of dynamical systems. The method of Lyapunov enables us to determine the stability properties of dynamical systems that cannot be solved explicitly by the closed forms. With constructing the suitable Lyapunov function the stability to the machines empower the stability of the overall interconnected system.

MSC 2010: 34D23; 34D20; 34C23

Keywords: power system stability analysis, transient stability, Lyapunov direct method

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Mahgoub transform method in stability research

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The traditional Fourier integral is where the Mahgoub Transform gets its name. Mohand Mahgoub developed the Mahgoub transform to make it easier to solve regular and partial differential equations in the time domain. The most practical mathematical methods for solving differential equations are often Fourier, Laplace, Elzaki, Aboodh, and Sumudu transforms. In order to solve differential equations, the Mahgoub transform and some of its essential features are also used.

The main aim of this study is to investigate the stability of the homogeneous and nonhomogeneous second order partial differential equations using the Mahgoub transform method.

MSC 2010: 34K20, 34K30, 34D04

Keywords: Mahgoub Transform Method, stability, partial differential equation

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Integral modification of Lupaş type operators

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This talk aims to modify Lupaş type operators to approximate integrable functions on $[0, \infty)$. Firstly, we propose Lupaş type operators which are called Lupaş-Jain operators and construct integral modification of Lupaş type operators. Then, we investigate the approximation results using some approaches. Lastly, we give a Voronovskaya-type asymptotic theorem.

MSC 2010: 41A36, 41A35, 41A25

Keywords: Lupaş-Jain operators, local approximation, Voronovskaya type theorem

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Multiple fixed point results for sum of operators and application

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In this talk, we are concerned with the study of existence, multiplicity, positivity and localization of solutions for abstract equations of the form: $Tx + Sx = x$, $x \in D$, where $(I - T)$ is a Lipschitz invertible mapping, S is a k -set contraction and D is a translate of cone of a Banach space. Then by making use of our theoretical results, general criteria on the existence of multiple positive solutions to the following singular generalized Sturm-Liouville multipoint boundary value problem are established:

$$\begin{aligned}
 -u''(t) &= h(t)f(t, u(t), u'(t)), \quad 0 < t < 1, \\
 au(0) - bu'(0) &= \sum_{i=1}^{m-2} a_i u(\xi_i), \\
 cu(1) + du'(1) &= \sum_{i=1}^{m-2} b_i u(\xi_i),
 \end{aligned} \tag{1}$$

where $a, b, c, d \in [0, \infty)$, $0 < \xi_1 < \xi_2 < \dots < \xi_{m-2} < 1$ ($m \geq 3$), $a_i, b_i \in [0, \infty)$ are constants for $i = 1, 2, \dots, m - 2$ and $\rho = ac + ad + bc > 0$.

MSC 2010: 47H10, 34B10, 34B24

Keywords: Fixed point, sum of operators, cone, Sturm-Liouville BVP, multiple positive solutions

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A note on some spaces induced by convex polyhedra and their isometry groups

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By the studies on metric space geometry it has been seen that some metrics and convex polyhedra are closely related. There are three essential methods in geometric investigations; synthetic, metric and group approach. The group approach treats isometry groups of a geometry and convex sets play a substantial role in indication of the isometry group of geometries. In this study, we investigate isometry groups of some Non-Euclidean spaces each whose unit ball is a truncated Catalan solid.

MSC 2010: 51B20, 51N25, 51F99, 51K05, 51K99, 52A15, 52B10

Keywords: Metric, truncated Catalan solids, isometry group

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Divided domains satisfy the weakly radical formula

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Throughout all rings are commutative and all modules are unitary. Let R be a ring and M be an R -module. A proper submodule N of M is called prime if whenever $rm \in N$, for some $r \in R, m \in M$ then $m \in N$ or $rM \subseteq N$. Also N is called weakly prime submodule if $rs m \in N$ implies $rm \in M$ or $sm \in N$ for some $r, s \in R, m \in M$. The notion of weakly prime submodule was introduced in [3]. If N is a proper submodule of an R -module M , then the prime radical of N , $rad_M(N)$, is defined as the intersection of all prime submodules containing N . Also, the weakly radical of N in M , $wrad_M(N)$, is the intersection of all weakly prime submodules of M containing N . Since every prime submodule is weakly prime, $wrad_M(N) \subseteq rad_M(N)$. The envelope of an R -module M , $E_M(N)$, is the set of all elements $x \in M$ where $x = rm$ with $r \in R, m \in M$ such that $r^k m \in N$ for some positive integer k . If for a submodule N of the module M , $wrad_M(N) = \langle E_M(N) \rangle$, then it is said that N satisfies the weakly radical formula (s.t.w.r.f).

If R is a commutative ring with identity whose prime ideals are linearly ordered, then $wrad_M(N)$ is weakly prime for any submodule N of an R -module M . In addition, if $\dim(R) \leq 1$, then R s.t.w.r.f. A domain R is called divided domain if every prime ideal of R is comparable to every principal ideal of R . We showed that divided domains satisfy the weakly radical formula. Also, we investigate under which conditions $wrad_M(N) = rad_M(N)$ for a submodule N of M .

MSC 2010: 13C99, 13A99, 13G05

Keywords: Prime submodule, weakly prime submodule, weakly radical formula, divided domains

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An overview of twistor theory

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Twistor theory studies physical information on a complex projective space which is called twistor space. It was first proposed by Roger Penrose in 1967. This theory has attracted great attention in mathematical physics recently and it approaches to quantum gravity. In this paper, we review twistor space of four-dimensional Minkowski space. In this order, we briefly mention about double cover of proper Lorentz transformation which is called $SL(2, \mathbb{C})$. Then, we study Clifford algebra and essential fundamental concepts identifying complex Minkowski space with Weyle spinor space, and finally we describe the geometric structure of twistor space and compare some properties between spacetime and twistor space.

MSC 2010: 32L25, 14D2, 83C60

Keywords: Twistor theory, Algebraic geometry, General relativity, Quantum theory, Clifford algebra

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QSPR investigation of leap Randić index of octanes

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Topological indices are used in QSPR studies modeling structural properties of molecules. Zagreb indices were used modeling π -electron energy levels of alternant hydrocarbons. Randić index was used modeling molecular branching which was defined after Zagreb indices. These both indices are the most used topological indices in chemical literature. Leap Zagreb indices have been defined recently as parallel to Zagreb indices by using 2-distance degree notion (connection number) in graph theory. Several QSPR studies show that leap Zagreb indices are possible tools in quantitative structure property relationship studies. Leap Randić index defined as “the product connectivity index” by using 2-distance degree notion (connection number). We firstly analyze the applicability of this index in view of QSPR studies by using some chemical properties of octanes. We show that leap Randić index give low correlation for the chemical properties of octanes. Also leap Randić index has low relationship with Randić index. We conclude that leap Randić index is not a possible tool for QSPR studies.

MSC 2010: 05C92, 05C09, 05C12

Keywords: : QSPR studies, leap Randić index, leap Zagreb indices, Connection number

On k-total distance degrees and related indices of graphs

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This study investigates the relationship between classical degree and recently defined k-distance degree, ve-degree and ev-degree concepts in graph theory. We firstly define the k-total distance degree notion and investigate its relation with Zagreb and Wiener polarity indices. One of the main relation is $W_3^*(T) = \frac{1}{2}M_1(T) + W_p(T)$ where $W_3^*(T)$, $M_1(T)$ and $W_p(T)$ denotes 3-total Wiener polarity index, the first Zagreb index and Wiener polarity index, respectively for any tree T .

MSC 2010: 05C92, 05C09, 05C12

Keywords: k-total distance degree, Wiener polarity index, k-total Wiener polarity index

Eigen gaps to decide on graph convolutional network architecture

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With the advent of artificial intelligence techniques, a large volume of deep learning-based methods have been developed for various real world applications. Among these methods, Graph Convolutional Network (GCN) [1] has been attracted much research attention; owing to its outstanding performances on graph related tasks, such as link prediction, node classification, and etc. The basic premise behind GCN is to propagate the features of nodes in a graph based on the graph's topological associativity and then transfer the resulting matrix by using a non-linear function, such as Rectified Linear Unit (ReLU) and a neural network to reduce the dimension of the matrix. More formally, Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph, \mathbf{A} is its adjacency matrix representation, and $\hat{\mathbf{L}}_{sym}$ be its Laplacian matrix to encode its associativity. Furthermore, let $\mathbf{F} \in \mathbb{R}^{n \times d}$ denotes the feature matrix of the graph. In essence, GCN uses the following propagation rules:

$$\mathbf{H} = ReLU(\hat{\mathbf{L}}_{sym} ReLU(\hat{\mathbf{L}}_{sym} \mathbf{F} \Theta^{(0)}) \Theta^{(1)}), \quad (1)$$

where $\Theta^{(0)}$ and $\Theta^{(1)}$ are trainable weight matrices. These weight parameters are trained using the following loss function:

$$\mathcal{L} = \underset{\Theta^{(0)}, \Theta^{(1)}}{\text{minimize}} \|\mathbf{A} - \sigma(\mathbf{H} \times \mathbf{H}^T)\|, \quad (2)$$

where σ denotes logistic sigmoid function. Later, it has been shown that using the second order proximity matrices, such as \mathbf{A}^2 , instead of the Laplacian matrix $\hat{\mathbf{L}}_{sym}$ as a propagation matrix can improve GCN's performance on link prediction tasks [2]. However, among many second order proximity matrices, which second order proximity should be used for better link prediction performance is still ambiguous and depends on the given dataset.

In this paper, to decide which proximity matrices should be used for which dataset, we argue that the gap of two leading eigenvalues can be a good indicator. Experimental results of these eigen-gap analyses show that we can use eigen-gaps for a priory knowledge to decide which proximity matrices should be used in GCN neural network architecture.

MSC 2010: 68R10, 68T05, 47J10

Keywords: Graph convolution network, embedding, eigenvalue gaps

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Spectral singularities of the Sturm-Liouville equations with a non-standard density function

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In this paper, we shall study the spectral properties of the Sturm-Liouville type differential operator with a non-standard density function. Spectral singularities of the Sturm-Liouville equations with a complex potential has been investigated in detail for the first time by Naimark [1]. Later, spectral singularities of the non-selfadjoint operators has been investigated for continuous and discrete cases. For detailed information, the interested reader may consult to the papers [2, 3, 4] and the references therein. For the first time in literature, in this paper, we make use of hyperbolic type representations of the fundamental solutions to obtain the quantitative properties of the spectral singularities. We also obtain the Jost function which is analytic in the left-complex half plane.

MSC 2010: 39A70, 47A10, 47A75

Keywords: Spectral singularities, spectral theory, resolvent operator, eigenvalues, discrete spectrum

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Bifurcation scheme of multi-layer flows on various wave velocities

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The hydrodynamics of a multi-layer flow have attracted the attention of many researchers because of the wealth and variety of waves which develop on multiphase interfaces. Besides experimental studies, the modelling of the problem has been widely studied in the literature but modelling of direct numerical simulations have been limited for moving faster wave families on various layer thicknesses for stationary film thickness. The bifurcation analyse of multi-layer flows with free surface is investigated by using approximate wave model. Calculations have been carried out to generate the bifurcation scheme which shows the families of various wave velocities. Examples of nonlinear wave shapes are illustrated at real-life values.

MSC 2010: 35Q30, 76A20, 76D05, 76D33

Keywords: Fluid dynamics, multi-layer flows, thin films, waves, bifurcation

On a solvable system of difference equations with constant coefficients

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The investigations of this study are concerned with the following three-dimensional system of difference equations

$$\begin{aligned}x_n &= \frac{y_{n-1}y_{n-2}}{x_{n-1}(\alpha + \beta y_{n-1}y_{n-2})}, \\y_n &= \frac{z_{n-1}z_{n-2}}{y_{n-1}(\gamma + \delta z_{n-1}z_{n-2})}, \quad n \in \mathbb{N}_0, \\z_n &= \frac{x_{n-1}x_{n-2}}{z_{n-1}(\epsilon + \zeta x_{n-1}x_{n-2})},\end{aligned}$$

where the parameters $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ and the initial values x_{-i}, y_{-i}, z_{-i} , $i \in \{1, 2\}$. In particular, we show that the general solution to system of difference equations are obtained in closed form and the asymptotic behavior of well-defined solutions to the aforementioned system is investigated by using the solutions obtained.

MSC 2010: 39A10, 39A20, 39A23

Keywords: Closed form, system of difference equations, long-term

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Examination of preservice mathematics teachers' written expression skills for geometric objects: Student diaries

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Education is seen as an important force that causes change and development in the world, and for this reason, a rapid change and development process is experienced in line with the needs in education as in every field. Developed countries have seen student diaries as a means of making written communication a part of mathematics learning, describing mathematical ideas, clarifying the areas that need improvement and developing positive attitudes towards mathematics. The written expression skills of preservice mathematics teachers on the subject of geometric objects within the scope of geometry learning were examined under the subtitles of being able to define, using concepts and using mathematical language. It was aimed to investigate preservice teachers' written expression skills by using the student diaries for geometric objects. This study, which is based on the qualitative research approach, was carried out at the 2021-2022 academic year. The participants of the study consisted of preservice teachers studying in the primary school mathematics teaching department of a state university in the Central Anatolia Region. In the research, qualitative data obtained from student diaries were analyzed by content analysis method. When the findings of written expression skills were examined, it was determined that the candidates mostly preferred to use verbal expressions and were weak in using mathematical language. On the other hand, it was seen that the candidates were able to associate the related concept with daily life.

MSC 2010: 97D30, 97D40, 97D60, 97G80

Keywords: Student diaries, written expression skills, geometric objects

Acknowledgement: This study was produced from the master thesis of the first author under the supervision of the second author.

Quasilinearization method for a generalized initial-value problem on the time scale

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Quasilinearization technique coupled with the method of lower and upper solutions provides an explicit analytic representation for the solution of nonlinear differential equation which yields pointwise upper and lower estimates for the solution of the problem.

In this paper, we have applied the well-known quasilinearization method to a given nonlinear differential equations on time scale. So, it was seen that similar results were obtained, parallel to the results obtained by applying this method for a nonlinear differential equation given by the classical derivative. Namely, we have constructed monotone sequences which converge uniformly and monotonically to the unique solution of the original problem under some conditions. The most important advantage of this method is that each element of the monotone function sequence is the solution of linear differential equations. Also, it has been shown that the convergence is quadratic.

MSC 2010: 34A12, 34A45, 34C11

Keywords: Quasilinearization, quadratic convergence, comparison theorem, time scale, lower and upper solutions

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Explicit and recursive formulas for Kirchhoff index of graphs via integer sequences such as generalized Fibonacci numbers

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Kirchhoff index of graphs can be computed by using either the eigenvalues of its Laplacian matrix or effective resistances between any two vertices. In both cases, finding explicit values is a complicated task in general. We use circuit reductions to derive recursive relations for various classes of graphs including ladder graphs [1] and prism graphs [2]. Solving the involved recursive relations enables us to obtain explicit formulas for the Kirchhoff indexes of those graphs. As a byproduct, we obtain recursive and explicit formulas for trigonometric sums. As an example, we derive the following equalities

$$\sum_{k=0}^{n-1} \frac{1}{1 + 2 \sin^2\left(\frac{k\pi}{n}\right)} = \frac{2nG_n^2}{G_{2n} - 2G_n},$$

where the sequence of integers G_n is defined recursively as follows:

$$G_{n+2} = 4G_{n+1} - G_n, \quad \text{if } n \geq 2, \text{ and } G_0 = 0, G_1 = 1.$$

For any positive integer n , we have

$$\sum_{k=0}^{n-1} \frac{1}{1 + 2 \sin^2\left(\frac{k\pi}{n}\right)} = \frac{n}{\sqrt{3}} \left[\frac{2}{1 - (2 - \sqrt{3})^n} - 1 \right].$$

MSC 2010: 05C12, 05C30, 05C50, 11B39, 11L03, 15A18, 94C15.

Keywords: Kirchhoff index, effective resistance, generalized Fibonacci numbers, prism graph, ladder graph, mesh graph

Acknowledgement: This work is supported by Abdullah Gul University Foundation.

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Approximation properties of a generalization of Szász-Beta operators

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In the present paper we examine the approximation properties of a generalization of Szász-Beta operators. We first analyze the approximation properties of the newly constructed operators by means of the Korovkin's theorem and then we give the rate of convergence results. Lastly we obtain a Voronovskaya type theorem.

MSC 2010: 41A25, 41A29, 41A30

Keywords: Szász-Beta operators, Korovkin's theorem, rate of convergence, Voronovskaya theorem

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New infinite classes of Laplacian borderenergetic graphs

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Let G be a graph of order n . G is said to be L -borderenergetic if its Laplacian energy is as same as the complete graph K_n , i.e. $LE(G) = 2(n - 1)$. In the paper [1], we proved 36 infinite classes of L -borderenergetic graphs. In this paper, we extend these classes and present 3 new infinite L -borderenergetic graphs Θ_1 , Θ_2 , and Θ_3 as composition of the cycle graphs under the operators product, union, join and complements. Let C_t be the cycle graph of order t and K_1 be the graph of single point. Then the graphs in the following 3 infinite classes are Laplacian borderenergetic.

1. $\Theta_1 = \{G = rK_1 \times \prod_{i=1}^k r_i C_{t_i} \text{ of order } n_1 = r \prod_{i=1}^k r_i t_i \text{ for } r = 1, 2, \dots, k = 1, 2, \dots, r_i = 1, 2, \dots, t_i = 1, 2, \dots, i = 1, 2, \dots\}$.
2. $\Theta_2 = \{G = rK_1 \times \bigcup_{i=1}^k r_i C_{t_i} \text{ of order } n_2 = r \sum_{i=1}^k r_i t_i \text{ for } r = 1, 2, \dots, k = 1, 2, \dots, r_i = 1, 2, \dots, t_i = 1, 2, \dots, i = 1, 2, \dots\}$.
3. $\Theta_3 = \{G = rK_1 \times \bigvee_{i=1}^k r_i C_{t_i} \text{ of order } n_3 = r \sum_{i=1}^k r_i t_i \text{ for } r = 1, 2, \dots, k = 1, 2, \dots, r_i = 1, 2, \dots, t_i = 1, 2, \dots, i = 1, 2, \dots\}$.

MSC 2010: 05C50, 05C76, 15A18

Keywords: Graph spectrum, Laplacian matrix, borderenergetic graph

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Some comparisons between the effects of planar and corrugated interfaces of a layer sandwiched between two half spaces on SH wave propagation

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In this study, the effects of periodic and planar boundary surfaces of the layer sandwiched between two elastic half spaces on the SH wave propagation are compared. The constituent materials of the half-spaces and the layer are assumed to be different, homogeneous, isotropic and incompressible. It is also assumed that corrugations of the interfaces are slowly varying functions of the distance in the direction of wave propagation. Numerical evaluation of the first branch of each dispersion relation has been performed for the following cases

- planar upper interface-periodic lower interface,
- periodic upper interface-planar lower interface,
- periodic upper interface-periodic lower interface

The variations of phase velocity and wave number of the waves with the direction of the wave propagation for each case mentioned above are presented graphically and the results are compared.

MSC 2010: 00A69, 35-XX, 35B20

Keywords: SH waves, corrugated surfaces, perturbation methods

Crumbling modules as injectivity domains

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Poor modules are introduced in [1] as an opposite of injectivity. The first wave in this line of research has focused mainly on the rings with no middle class (over which every right module is either poor or injective). In one of such studies, the authors related the existence of semisimple poor modules to the coincidence of the class of all crumbling modules (exactly locally Noetherian V -modules) and the class of all semisimple modules (see [2]). [3] is one of the papers that consider rings that can have more than one middle class.

In this talk, we focus on the case when there is only one middle class of injectivity domains. Especially, we investigate the properties of the rings that have the class of crumbling modules as their only middle class of injectivity domains.

MSC 2010: 16D50, 16D70, 16D80

Keywords: Injective module, poor module, modest module, SSI-ring, V -ring, WV -ring, Noetherian ring

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Lacunary statistical convergence of multiset sequences

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Statistical convergence developed rapidly after being defined independently by Fast and Steinhaus in 1951 and was studied in many fields. One of them, lacunary statistical convergence, was defined by Fridy and Orhan in 1993. On the other hand, although there are various studies on multisets, which are sets that can repeat elements, the convergence of multisets was defined by Pachilangode in 2021.

In this study, lacunary statistical convergence of multiset sequences is examined and related examples and theorems are given.

MSC 2010: 40G15, 40A35

Keywords: Multiset sequences, statistical convergence, lacunary sequence

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The nabla z-transform

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In this work, we propose the nabla z-transform, denoted by Z_{∇} , for solving linear nabla difference equations. Through this novel approach, we derive the nabla z-transform of some basic functions that compose parts of the mentioned difference equations and use these results to solve some nabla difference initial value problems.

MSC 2010: 34N05, 26D10, 26D15

Keywords: Nabla z-transform, difference equation

Artinian analogues of some Noetherian modules over complete semi-local commutative Noetherian rings

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The goal of this research is to come up with a version of some well-known lemmas that applies to Artinian modules. We create an extension of Matlis duality that applies to a complete semi-local Noetherian ring to prove the Artinian case; this allows us to switch back and forth between Noetherian and Artinian modules. This technique is combined with the completeness of R (the ring over which we construct modules) in relation to R -module A to demonstrate how various Artinian module findings can be inferred from well-known classical Noetherian conclusions. Matlis' classical duality was created for a complete local Noetherian ring at first. We take use of this fact.

MSC 2010: 13E10, 13E05

Keywords: Artinian rings and modules, finite dimensional algebras, Noetherian modules

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Upper bounds for the blow up time for the Kirchhoff-type equation

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In this presentation, we consider the Kirchhoff type equation with variable exponent. This type problem has been widely used in many mathematical models of various applied sciences such as flows of electro-rheological fluids, thin liquid films, etc. We prove the upper bound for blow up time under suitable conditions.

MSC 2010: 35B44, 35L10, 46E35

Keywords: Blow up, Kirchhoff-type equation, variable exponent

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New oscillation/ nonoscillation criteria for impulsive differential equations with discontinuous solutions

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There is a vast amount of literature on oscillation/ nonoscillation of the classical self adjoint ordinary differential equation

$$(p(t)x')' + q(t)x = 0. \quad (1)$$

In particular, Moore [2] and Hille [3] obtained some oscillation/ nonoscillation criteria involving the integrals

$$p_1(t) = 1 + \int_a^t \frac{ds}{p(s)}, \quad p_0(t) = \int_t^\infty \frac{ds}{p(s)}.$$

In the present work, our aim is to obtain generalizations of the Moore's and Hille's results, and some of their improved modifications for impulsive differential equations having discontinuous solutions.

MSC 2010: 34B15, 34B37, 34A37

Keywords: oscillation/ nonoscillation, impulsive, discontinuous

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A first countable Hausdorff topology induced by a quasi-uniformity

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A quasi-uniform structure on a nonempty set X is a filter $\mathfrak{U}_q \subseteq P(X \times X)$ satisfying the following conditions:

(QU1) Each element $U \in \mathfrak{U}_q$ includes the set $D = \{(x, x) \mid x \in X\}$,

(QU2) For every $U \in \mathfrak{U}_q$, there exists $V \in \mathfrak{U}_q$ such that $V \circ V \subseteq U$.

The pair (X, \mathfrak{U}_q) is called a quasi-uniform space.

In this talk, we define a first-countable Hausdorff topology on the space (X, \mathfrak{F}, t) induced by a quasi-uniformity, where $\mathfrak{F} : X \times X \rightarrow \Delta$ is a function satisfying the following conditions;

(U1) $\mathfrak{F}_{xx} = \epsilon_0$

(U2) $\mathfrak{F}_{xz}(k_1 + k_2) \geq t(\mathfrak{F}_{xy}(k_1), \mathfrak{F}_{yz}(k_2))$, for all $k_1, k_2 > 0$

(U3) $\sup\{t(a, a) \mid a < 1\} = 1$

Here Δ denotes the set regarding distribution functions

MSC 2010: 54E15, 54E70, 54E99

Keywords: Probabilistic metric space, quasi-uniform space, distribution functions

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Almost continuity and almost compactness in graded ditopological texture spaces

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In the theory of graded ditopology, openness and closedness are given by independent grading functions instead of elements of a texture as in ditopological case. So, in this structure, the notion of ditopology on a texture is fuzzified in a sense.

In this talk, the concepts of almost (bi)continuous difunction and almost (di)compactness in graded ditopological texture spaces are introduced and some of their properties are investigated. Also the relation between almost (di)compactness in graded ditopological texture spaces and both almost (di)compactness in ditopological texture spaces and near (di)compactness in graded ditopological texture spaces are studied.

MSC 2010: 54D30, 54A05, 54A40

Keywords: Almost compactness, almost continuity, graded ditopology

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Nonnegative bias reduction methods for density estimation using scaled inverse chi-squared kernel estimator

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The problem of estimating the unknown probability density function (pdf) of a univariate random variable X has been the subject of research for a long time. Standard kernel estimators were developed primarily for the densities with unbounded support using a symmetric pdf as a kernel. However, symmetric kernel estimators cause a boundary bias problem when used to fit densities with compact or semi-infinite support. This problem occurs because the kernel assigns positive weights outside the support when smoothing applied near the boundary. Thus, the asymmetric kernel estimators have been proposed as a good alternative way for avoiding these boundary effects. It is known that, the bias of asymmetric kernel estimators is $O(b)$ as $b \rightarrow 0$ under sufficient differentiability conditions. In this work, two classes of multiplicative bias correction (MBC) methods, originally proposed for density estimation using symmetric second order kernels, are extended to asymmetric Scaled Inverse Chi-Squared (SCI) density estimator to achieve the rate improvements. It is shown that, under sufficient smoothness of the true density, both MBC techniques reduce the order of magnitude in bias, while the order of magnitude in variance remains unchanged. In this case, the bias convergence leads to faster mean squared error and mean integrated squared error convergence rates for both MBC estimators compared to the classical SCI kernel estimator. Simulation studies are performed to compare the performance of the two MBC estimators and classical SCI estimators and then a real data set is presented to illustrate the findings.

MSC 2010: 62G05, 62G07, 41A25

Keywords: Density estimation, asymmetric kernel estimators, boundary bias problem, multiplicative bias correction method

Stability, boundedness and existence of periodic solutions of some fourth order non-linear differential equations with multiple delays

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The aim of this work is to investigate stability, boundedness and existence of periodic solutions of some fourth order nonlinear differential equations with multiple delays by using the Lyapunov-Krasovskiĭ functional approach. We gave an example to show applications of the given results. The given results are new and have contributions to the qualitative behaviors of solutions of higher order delay differential equations.

Subject Classification: 34C25, 34D05, 34D20

Keywords: Delay differential equation, stability, boundedness, existence of periodic solution, fourth order

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On stability of linear difference systems

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Systems of linear difference equations being equivalent to a vector equation are considered and when the coefficient matrix is constant, the solutions of the corresponding homogeneous equations are shown to be varying according to the multiplicities of the eigenvalues of the coefficient matrix being greater than one or not. Furthermore, the developed techniques in determining these solutions are employed to describe and illustrate various stability criteria of solutions of the above explained systems.

MSC 2010: 34N05, 26D10, 26D15

Keywords: Linear difference equation systems, stability

Cofinitely ss -lifting modules

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An R -module M is named cofinitely semisimple lifting or briefly cofinitely ss -lifting if for each cofinite submodule S of M , M has a decomposition $M = U' \oplus V$ such that $U' \subseteq S$ and $S \cap V \subseteq Soc_s(V)$. In this study, equivalent conditions to this definition are given. In addition, the basic features of this concept defined in this article are examined.

MSC 2010: 16D10, 16D60, 16D99

Keywords: Cofinitely ss -lifting module, cofinitely weak ss -lifting module, ss -lifting module, weak ss -lifting module

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An operator-theoretic setting of singly-generated invariant subspaces in the polydisc

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The structure of invariant subspaces of the Hardy space on the unit disc are very well known; however, in several variables, the structure of the invariant subspaces of the classical Hardy spaces is not yet fully understood. In this talk, we completely characterize the structure of singly-generated invariant subspaces of the Hardy space on the polydisc using the well-known Beurling-Lax-Halmos Theorem. As a result of our characterization, it is also obtained the structure of Beurling-type invariant subspaces in the polydisc.

MSC 2010: 47A15, 32C15

Keywords: Hardy spaces, invariant subspaces, operator valued function, polydisc

Optimal control on the MHD mixed convection flow with transverse magnetic field

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Optimal control of the steady, laminar, magnetohydrodynamics (MHD) mixed convection flow of an electrically conducting fluid is considered under the effect of transverse magnetic field in a square duct. The viscous and Joule dissipations are included and the flow is driven by a constant pressure gradient. The tripled nonlinear set of momentum, induction and energy equations are solved in dimensionless form by using mixed finite element method (FEM) with the implementation of Newton's method for nonlinearity with discretize-then-linearize approach. Accordingly, FEM solutions are obtained for various values of the problem parameters to ensure the efficiency of the underlying scheme.

This study aims to investigate the problem of controlling the steady flow by using the physically significant parameters of the problem as control variables [1, 2] in the case of a mixed convection flow [3]. In this respect, classification of the type of the convection, forced or free, is achieved by controlling the Grashof number (Gr). Besides, single and pairwise controls with Hartmann number (M), Prandtl number (Pr) and magnetic Reynolds number (Rm) are also used to regain the prescribed fluid behaviors and required magnetic field. Control simulations are conducted with Sequential-Least-Squares-Programming (SLSQP) algorithm in the optimization.

MSC 2010: 65N30, 90C53, 93C20

Keywords: MHD, Optimal control, FEM

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A logarithmic barrier interior point method for linear programming

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Interior point methods are one of the efficient methods developed to solve linear, semidefinite and nonlinear programming problems. In this paper, we particularly propose a logarithmic barrier interior point method for solving a linear constrained programming problem, the logarithmic barrier function used is given by $-\sum_{i=1}^n \ln(x_i)$ where $r > 0$. We ensure the convergence when r tends to zero. Various approaches are developed to calculate the step size. In our work we present a comparative numerical study between a majorant function technique and a tangent technique for calculate the step size t along a Newton descent direction d .

The linear programming studied in this paper is defined as follows:

$$\begin{cases} \min b^t z \\ A^t z \geq c \\ z \in \mathbb{R}^n \end{cases} \quad (\text{P})$$

In which: $A \in \mathbb{R}^{m \times n}$ is a full rank matrix ($m < n$), $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$.

MSC 2010: 90C05, 90C25, 90C51

Keywords: Linear constrained programming, interior point methods, search line, majorant function, tangent technique

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On automorphisms of symmetric polynomials of noncommutative algebras

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A polynomial in the set $X_n = \{x_1, \dots, x_n\}$ of (nonnecessarily commutative) variables is called symmetric if it is preserved under any change of elements of X_n . It is well known that the set of such polynomials form the algebra of invariants of the symmetric group in the corresponding relatively free algebra generated by X_n . In the current study, we consider the algebra of symmetric polynomials of some certain relatively free noncommutative algebras and describe their automorphisms.

MSC 2010: 17B01, 17B30, 17B40

Keywords: Automorphism, noncommutative, symmetric polynomial

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The Voronovskaja type theorem for a class of Kantorovich type operators associated with the Charlier polynomials I

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In this study, initially, we give a generalization of Kantorovich type linear positive operators that includes Charlier polynomials. Secondly, we present the moments of these operators and then prove their uniform convergence using Bohman-Korovkin's theorem for convergence of these operators towards the identity operator. In addition to the above mentioned, we give the Voronovskaja type formula with the classic method to measure the effectiveness of approximation.

MSC 2010: 41A10, 41A25, 30E10

Keywords: Charlier polynomials, Voronovskaja type theorem, Kantorovich type operators, positive operators

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New qualitative criteria to certain vector differential equations of second order

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This paper considers a certain class of nonlinear vector differential equations of second-order. Qualitative properties of solutions of the considered equations called stability, asymptotic stability, uniform stability, boundedness and uniformly boundedness of solutions are investigated. Four new theorems related to the mentioned qualitative concepts are proved by using the second method of Lyapunov. Examples are given to show applicability of the results. By this paper, we generalize and improve some results that are available in the literature.

Subject Classification: 34K20, 34K06, 34K40

Keywords: Differential equations, stability, second order, Lyapunov functional, boundedness

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A new class of $m - (p, q)$ -Bernoulli polynomials

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The main object of this talk is to give an interesting formula for calculating a new class of $m - (p, q)$ -Bernoulli numbers and polynomials which is presented on three-term recurrence formula. This class of numbers and polynomials result from the Gaussian hypergeometric function and its properties. We provide some explicit formulas in terms of m -Stirling numbers and the weighted Stirling numbers of the second kind.

MSC 2010: 11B68, 11B73, 33C05

Keywords: Bernoulli numbers, Stirling numbers, Gaussian hypergeometric function, three-term recurrence formula

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A fully discrete scheme for singularly perturbed semilinear integro-differential equations with two integral boundary conditions

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This paper purposes to present a discrete scheme for the singularly perturbed semilinear mixed integro-differential equation including integral boundary conditions. Initially, some analytical properties of the solution are given. Then, using the composite numerical integration formulas and implicit difference rules, the finite difference scheme is established on a uniform mesh. Error approximations for the approximate solution and stability bounds are investigated in the discrete maximum norm. Finally, a numerical example is solved to show ε -uniform convergence of the suggested difference scheme.

MSC 2010: 34D15, 65L10, 65L12, 65L20, 65L70

Keywords: Discrete scheme, error bounds, integral boundary condition, integro-differential equation, singular perturbation

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On the analysis of a model with Holling type-III functional response consisting of super predator, intermediate predator and prey

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In this study, a model consisting of an upper predator, intermediate predator and prey populations is discussed.

$$\frac{dR}{dT} = rR \left(1 - \frac{R}{K} \right) - \frac{\alpha_1 R^2 N}{\beta_1 + R^2} - \frac{\alpha_2 R^2 P}{\beta_2 + R^2} \quad (1)$$

$$\frac{dN}{dT} = \frac{e_1 \alpha_1 R^2 N}{\beta_1 + R^2} - \frac{\frac{\alpha_3}{1+R} N^2 P}{\beta_3 + N^2} - \gamma_1 N^2 - \eta_1 NP - D_1 N \quad (2)$$

$$\frac{dP}{dT} = \frac{e_2 \alpha_2 R^2 P}{\beta_2 + R^2} + \frac{e_3 \frac{\alpha_3}{1+R} N^2 P}{\beta_3 + N^2} - \gamma_2 P^2 - \eta_2 NP - D_2 P \quad (3)$$

In order to address the diversity in the behaviour of these populations, the upper predator may feed on both intermediate predator and prey. Earlier studies had examined Holling type 1 and Type 2 functional response modeling in an ecosystem consisting of these three species. In this study, Holling Type 3 functional response is considered where predation saturates as the number of prey species exceeds a certain number. The existence of possible equilibrium points of the system and their local stability are investigated. Numerical simulations are conducted to explain the theoretical results.

MSC 2010: 34Dxx, 37N25, 34Cxx

Keywords: Prey switching, functional response, Holling Type III, equilibrium point, global stability

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On the Perrin hybrid quaternions

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A quaternion is defined [3] by $q = a_0 + a_1i + a_2j + a_3k$, where a_0, a_1, a_2, a_3 are real numbers and i, j, k are quaternionic units. Several authors studied on different quaternions and their generalizations, some of which can be found in [1, 2, 4]. A new set of numbers called hybrid numbers are defined in [5] as a generalization of dual numbers, complex numbers and hyperbolic numbers. Since then, the hybrid numbers have been a topic of interest, and some new classes of quaternions and sequences have been studied in [1, 6, 7]. Inspired by these works, we define a new type of quaternions called Perrin hybrid quaternions. We give recurrence relations, Binet-like formulas, generating functions, exponential generating functions and some properties for these quaternions. Then we also define an associate matrix for these quaternions. By the means of this matrix, we also give several identities of these quaternions. We obtain matrix representations of these quaternions by Matlab and calculate determinants of quaternion matrices with Matlab applications.

MSC 2010: 11B39, 11K31, 11Y55, 11R52

Keywords: Recurrence sequence, Perrin number, hybrid number, quaternion

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Introduction to Grey models

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Grey system theory was introduced to find solutions to problems involving small sample and weak information. Its application areas have five components, namely Grey relational analysis (GRA), Grey modelling, Gray prediction, Grey control and Grey decision making. In the process of creating the grey prediction model, the original data is accumulated into a new data column with strong regularity and a new model is created. Then, the reduction model is created from this new data column by inverse operation and the final prediction model is obtained. The GM(1,1) model is the most widely used grey forecasting model. The grey model was developed to make inferences for future periods and to predict them based on the values of data from past periods. Today, many quantitative techniques are used for the estimation of time series. However, the important thing here is to keep the error rates that occur as a result of the predictions made by the techniques at a minimum level. In this sense, sometimes by modifying the error terms and sometimes by combining different techniques, it is tried to reduce the error rates and to obtain more accurate estimations. Because the plans made for the future and the policies that can be determined gain meaning only with these predictions. The lower the error rate, the higher the performance of the prediction model will be. With these in mind, GM(1,1) is one of the most commonly used Grey forecasting models. This model is a time series prediction model consisting of first order differential equations. The minimum number of data should be four to create the GM(1,1) model. In general, GM(n,m) refers to the grey model. n is the number of nth order differential equations, m is the number of variables.

MSC 2010: 60G25, 34B60, 68U01

Keywords: Grey system theory, grey prediction, GM(1,1), grey control

A generalized form of ρ -statistical convergence of interval numbers

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Statistical convergence, which has been studied in many fields after the 1950s, was studied together with a $\rho = (\rho_n)$ sequence by Cakalli in 2017 [2] and this new concept was named ρ -statistical convergence. Later on, Aral, Kandemir and Et [1] studied ρ -statistical convergence for set sequences. Gumus defined ρ -statistical convergence for interval numbers.

In this study, we examine the concept of ρ -statistical convergence for interval numbers in a more general way using the $p = (p_k)$ sequence of positive real numbers.

MSC 2010: 40G35, 40A15

Keywords: Statistical convergence, ρ -statistical convergence, interval numbers

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Arithmetical reduction of classification problems of singular K3-surfaces

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It has become quite clear that the classical approach to the classification problems of singular models of $K3$ -surfaces (such as sextics and quartics) based on the defining equations is bound to fail: even when the classification is known, it requires a tremendous amount of work to find an explicit equation of any given curve. Therefore, we use the more modern approach based on the $K3$ -surfaces suggested in [6], [1], [2], which reduce the classification problem to a certain arithmetical problem concerning lattice extensions. Application of this $K3$ -theoretical approach has been limited by the huge number of classes and the necessity to compute the infinite orthogonal groups of indefinite lattices. We overcome this difficulty by the principal novelty of the computer aided usage of the Miranda—Morrison theory ([3], [4], [5]) which reduce the computation to the finite discriminant forms and make the whole problem almost algorithmical.

MSC 2010: 14H45, 14J28, 14H30, 14J10

Keywords: $K3$ -surface, simple singularity, plane sextic, spatial quartic

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Advanced refinements of Berezin number inequalities

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For a bounded linear operator A on a reproducing kernel Hilbert space $\mathcal{H}(\Omega)$, with normalized reproducing kernel $\widehat{k}_\eta := \frac{k_\eta}{\|k_\eta\|_{\mathcal{H}}}$, the Berezin symbol and Berezin number are defined respectively by $\widetilde{A}(\eta) := \left\langle A\widehat{k}_\eta, \widehat{k}_\eta \right\rangle_{\mathcal{H}}$ and $\text{ber}(A) := \sup_{\eta \in \Omega} \left| \widetilde{A}(\eta) \right|$. A simple comparison of these properties produces the inequality $\text{ber}(A) \leq \frac{1}{2} \left(\|A\|_{\text{ber}} + \|A^2\|_{\text{ber}}^{1/2} \right)$ (see [4]). In this paper, we prove further inequalities relating them, and also establish some inequalities for the Berezin number of operators on reproducing kernel Hilbert spaces.

MSC 2010: 47A30, 47A63

Keywords: Berezin symbol, Berezin number, reproducing kernel Hilbert space, positive operator

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On designing discrete-time games from continuous-time models

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A continuous-time game is a mathematical concept which is used in game theory where each player's course of action depends on a continuous dynamical process emerged from real and non-real life forms [1]. In other words, it extends the notion of a discrete-time game, a game in which each player's move is determined by a finite set of pure strategies in discrete-time. Hence, a continuous-time game model allows one to investigate more general sets of pure strategies, which may result in uncountably infinite. In this work, we study the connection between these two game models and design discrete-time platforms from the given continuous-time playing ground to achieve game ending scenarios for pursuit-evasion type game models in discrete-time. In recent literature, designing such game structures are studied in [2, 3, 4, 5, 6, 7] to deal with some real-world problems.

MSC 2010: 91A06, 91A10, 91A24, 91A50.

Keywords: Game theory, continuous-time system, discrete-time system, control, finite set

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The detour index of graph products

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The object of this paper is to investigate the detour index of some graph products. The detour index of a connected graph is defined as the sum of the detour distances (lengths of longest paths) between all pairs of vertices of the graph. The detour index is a well-known topological index, which is used in quantitative structure-activity relationship (QSAR) studies. In this paper, we establish expressions for the detour index of join and corona of some products of graphs.

MSC 2010: 05C12, 05C76, 05C30

Keywords: Detour index, join graphs, corona graphs, quasi-corona graphs, topological index, chemical graph theory

On commutant hypercyclicity and commutant transitivity of some operators

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Linear dynamics is a well-established topic of research that has experienced a great development in recent decades with two monographs [1] and [2]. In this presentation we will study commutant hypercyclicity and commutant transitivity of operators. We will introduce some operators which are not commutant hypercyclic and next we will show that some operators are commutant hypercyclic but not commutant transitive. Furthermore, we will present the sufficient and necessary conditions for a vector to be a commutant hypercyclic vector.

MSC 2010: 47A16, 47B37, 47B99

Keywords: Commutant hypercyclicity, commutant transitivity, Hilbert spaces, operators

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On fractional variable-order neural networks based on the Caputo derivative

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Because of its memory and heredity qualities, the results of merging neural networks with fractional calculus are very outstanding [1]. Yet, variableorder fractional operators were just recently established and formally described; however, due to their capacity to build evolutionary governing equations, these operators have been effectively employed to represent complex significant problems [2][3]. To that end, we investigate the existence, uniqueness, and stability of variable-order fractional neural network solutions in this paper. First, the Caputo variable-order fractional derivative will be included in the fractional-order neural networks. The existence and uniqueness of the proposed neural networks will next be demonstrated using fixed point theorems. Furthermore, we will use direct analytical approaches to examine Ulam–Hyers stability for variable-order fractional operators. Finally, two-dimensional example and simulations employing the Adams-Bashforth-Moulton schema [4] are shown to demonstrate our theoretical results.

MSC 2010: 26A33, 34K20, 45G15

Keywords: Variable-order fractional neural networks, Caputo variable-order fractional derivative, Fixed point theorem, Ulam-Hyers stability

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Every global action defines a local equivalence relation

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A global action is the algebraic analogue of a topological space. This structure was introduced in first place by A. Bak [1, 2] as a combinatorial approach to K-Theory and the concept was later generalized by Bak, Brown, Minian and Porter to the notion of groupoid atlas [3]. The notion of local equivalence relation was introduced by Grothendieck and Verdier [4] in a series of exercises presented as open problems concerning the construction of a certain kind of topos. It was investigated further by Rosenthal [5, 6] and more recently by Kock and Moerdijk [7] and in general case Brown and İcen [8]. In this paper we investigate that the notion of global action on a topological space defines a local equivalence relation.

MSC 2010: 18F25, 55N15, 55N99

Keywords: Global actions, local equivalence relation, action

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Applications of the piecewise derivative to real-world problems

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In this study, we deal with the usage of the piecewise derivative newly introduced by Atangana and Araz to simulate some real-world problems. This concept has been used to model different processes in different time periods and has been successfully applied in many disciplines such as chaos, epidemiology and mathematical biology. It can be seen that some previously simulated problems can be modified again with the help of piecewise derivatives. For example, the Van-Der Pol equation has been used to model heart rhythm before and many studies have been done by researchers. Imagine a person having a heart attack, and while before the heart attack this person's heart rhythm is normal, during a heart attack the heart rhythm will increase over time. If the treatment fails, this person will die and his heart will stop. While the Van-Der Pol equation cannot model these three processes by itself, this process can be described with the concept of piecewise derivative. This study will present some applications regarding the usability of the piecewise derivative concept in such processes. Since there are many problems in nature that exhibit different behaviors at such different times, we strongly believe that this new concept will open new doors in applications.

MSC 2010: 35A23, 65P20, 92D30

Keywords: Piecewise differential and integral operators, chaos, epidemiology

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A C^1 approximation on differential inclusions

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For a given set-valued map $F : [0, T] \times \mathbb{R}^n \rightarrow 2^{\mathbb{R}^n} \setminus \{\emptyset\}$ we consider the following differential inclusion

$$\dot{x}(t) \in F(t, x(t)),$$

where $T > 0$.

In this study, we present some existence results about approximations of classical (i.e. continuously differentiable) solutions for the differential inclusion with initial condition $x(0) = x_0$. For this, we make use of Carathéodory type solutions, i.e. absolutely continuous functions x that satisfies the differential inclusion with the initial condition almost everywhere on $[0, T]$, and some results related to selection theorems for set-valued maps.

MSC 2010: 34A60, 34A12, 34A45

Keywords: differential inclusion, set-valued map, approximation, classical solution

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On solutions of three-dimensional system of difference equations

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In this study, we show that the system of difference equations

$$\begin{aligned}x_n &= \frac{x_{n-2}y_{n-3}}{y_{n-1}(a + bx_{n-2}y_{n-3})}, \\y_n &= \frac{y_{n-2}z_{n-3}}{z_{n-1}(c + dy_{n-2}z_{n-3})}, \quad n \in \mathbb{N}_0, \\z_n &= \frac{z_{n-2}x_{n-3}}{x_{n-1}(e + fz_{n-2}x_{n-3})},\end{aligned}$$

where the parameters a, b, c, d, e, f and the initial values $x_{-i}, y_{-i}, z_{-i}, i \in \{1, 2, 3\}$ are non-zero real numbers, can be solved in the explicit form. In addition, we define the forbidden set of the initial conditions by using the obtained formulas. Finally, we get periodic solutions of aforementioned system.

MSC 2010: 39A10, 39A20, 39A23

Keywords: Explicit form, System of difference equations, forbidden set

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Stability and bifurcation analysis of a COVID-19 mathematical model on Erdős Rényi network

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In this study, a mathematical model consisting of conformable fractional order differential equations is proposed to describe COVID-19 dynamics. By applying a discretization procedure based on the use of piecewise constant arguments to the model a system of difference equations is obtained. By using the Schur-Cohn criterion, necessary and sufficient conditions are obtained for asymptotic stability of a positive equilibrium point of the discrete system. Bifurcation analysis shows that Neimark-Sacker bifurcation occurs around the positive equilibrium point due to the changing the parameter d and e in the system. Moreover calculating maximum Lyapunov exponents exhibit chaotic structures in discrete dynamical system. In addition, the COVID-19 mathematical model consisting of healthy and infected populations is also studied on the Erdős Rényi network. If the coupling strength reaches the critical value then transition from nonchaotic to chaotic state is observed in complex dynamical networks. Finally, all theoretical results are supported by numerical simulations.

MSC 2010: 39A28, 39A30, 92B20, 37N25

Keywords: Piecewise constant arguments, difference equation, stability, Neimark-Sacker bifurcation, complex network

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Different bounds for degree Kirchhoff index

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Let G be a simple, connected (molecular) graph with the vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. The distance d_{ij} is the number of edges of shortest path between vertices v_i and v_j in G . If G is a connected (molecular) graph then the minimum and maximum degree will be represented by δ and Δ , respectively. The degree Kirchhoff index of G is explained as a new index $Kf'(G) = 2m \sum_{i=0}^{n-2} \frac{1}{\rho_i}$ in [4] and [7]. In this paper, some new bounds for degree Kirchhoff index.

Let G be a connected graph of order $n > 3$ and t spanning trees then

$$Kf(G) \geq \frac{2m}{\Delta + 1} + \frac{2m(n-1)}{n-3} \left(\frac{(\Delta + 1)^{\frac{n-3}{n-1}}}{(nt(G))^{\frac{n-2}{n-1}}} - \frac{1}{(nt(G))^{\frac{1}{n-1}}} \right). \quad (1)$$

with equality holding if and only if $G \cong K_n$.

MSC 2010: 05C10, 05C92

Keywords: Graph, degree, Kirchhoff index

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Novel delta and nabla Pachpatte type inequalities via convexity

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Nabla Pachpatte type dynamic inequalities are obtained via convexity to unify the results established for the special cases, which are continuous and discrete cases. By this unification, delta Pachpatte type dynamic inequalities are established as well. Some of the obtained inequalities are nabla counterparts of their delta versions while the others are new even for the discrete, continuous and delta cases.

MSC 2010: 34N05, 26D10, 26D15

Keywords: Time scale calculus, Pachpatte inequality, Hardy inequality, convexity

Diamond-alpha Pachpatte type dynamic inequalities

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Diamond-alpha Pachpatte type dynamic inequalities are obtained via convexity to unify the results established for the special cases, which are delta and nabla calculi. The obtained inequalities are novel not only for the diamond-alpha calculus but also for these special cases. Moreover the method of the proof of the theorems is different from the proofs in the special cases because the Integration by Parts Formula and the Fundamental Theorem of Calculus, which are the main tools for the proofs of the theorems in the delta and nabla approaches, do not exist in the diamond-alpha calculus.

MSC 2010: 34N05, 26D10, 26D15

Keywords: Diamond-alpha time scale calculus, Pachpatte inequality, Hardy inequality, convexity

On oscillation of solutions of advanced differential equations

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In this study, the linear advanced differential equations

$$x'(t) - p(t)x(\tau(t)) = 0$$

and nonlinear advanced differential equations

$$x'(t) - p(t)f(x(\tau(t))) = 0$$

are investigated, where $p, \tau \in C([t_0, \infty), \mathbb{R}^+)$, $\tau(t) \geq t$ and $\lim_{t \rightarrow \infty} \tau(t) = \infty$ and $xf(x) > 0$ and $f \in C(\mathbb{R}, \mathbb{R})$ for $x \neq 0$. Furthermore, the oscillatory conditions of these equations are presented. Finally, some examples are given to illustrate the results.

MSC 2010: 34C10, 34G20, 34K11

Keywords: Advanced differential equations, monotone argument, nonmonotone argument, nonoscillatory solution, oscillatory solution

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Factorization of a statistical matrix

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We define a square matrix with entries including q -integers and the powers of the indeterminates x and y . We see that the well known Helmert matrix can be obtained for some values of q , x and y . We are interested in the LU factorization of defined matrix since LU decomposition of a matrix gives much information about its structural properties. We also examine some properties of these matrices corresponding to some special values of q , x and y .

MSC 2010: 15A23, 05A30, 65F35

Keywords: Factorization of matrices, q -calculus, matrix norms

Roth's theorem on arithmetic progressions

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An *arithmetic progression of length k* (k -AP) is a sequence of k -numbers such that each difference between two consecutive terms is the same constant. Finding long arithmetic progressions in certain subsets of integers is at the center of mathematics in the last century. In this talk, we will first give a quick survey of remarkable results on this topic. In particular, we will focus on Roth's theorem on arithmetic progressions. Finally, we will also talk about recent significant results.

MSC 2010: 11B25, 11B75, 11L03

Keywords: Arithmetic progression, Roth's theorem, upper density

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A note on nilpotent units in commutative group rings of direct product groups

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Let $V(RG)$ display the normalized unit group of group ring RG of a group G over a ring R . G -nilpotent units in a commutative group ring has been defined in [3]. In this talk, some necessary and sufficient conditions for a normalized unit group in a commutative group ring of a direct product group $G \times H$ to consist only of $G \times H$ -nilpotent units have been explained and especially some results which are related to groups $G \times C_3$ and $G \times C_4$ have been introduced where C_3 and C_4 are cyclic groups of orders 3 and 4 respectively. In this context, we can say that the study extends the results in [3]. At the end, some open problems which are inspired by nilpotent units are served as future works.

MSC 2010: 16S34, 16U60, 20K10, 20K20, 20K21

Keywords: Nilpotent, unit, group ring, direct product group

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On the numerical solution of the fractional integro-differential equation using shifted Chebyshev polynomials of the third kind

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The presented work aims to provide a new numerical approach for linear fractional integro-differential equations based on collocation method. A Gauss collocation method is developed with the use of shifted Chebyshev polynomials of the third kind to approximate the solution. The fractional derivative is described in the Caputo sense. Convergence and error analysis are discussed. Some examples are presented to confirm the theoretical results and prove the efficiency and reliability of the numerical approach.

MSC 2010: 34A08, 26A33, 33C47

Keywords: Fractional integro-differential equation, shifted Chebyshev polynomials of the third kind, spectral collocation method, Caputo fractional derivative

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On semicommutative rings in term of nilpotent elements

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Semicommutative rings are a generalization of reduced rings, and thus, nilpotent elements play an important role in these rings. There are many examples of rings with nilpotent elements which are semicommutative ring. We introduce the notion of nilpotent-semicommutative rings which is a generalization of semicommutative rings. A ring R said to be a nilpotent-semicommutative ring if $\forall a, b \in R, ab = 0$ implies $aN(R)b = 0$. where $N(R)$ stands for the set of nilpotents of R . A lot of properties of nilpotent-semicommutative rings are introduced and many known results on semicommutative rings are extended.

MSC 2010: 16S36, 16N40, 13F20

Keywords: nilpotent semicommutative ring, semicommutative ring, reduced ring

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Boundedness of sum of generalized weighted composition operators between weighted spaces of analytic functions

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Let $\mathcal{H}(\mathbb{D})$ be the space of analytic functions on the unit disc \mathbb{D} and let $\mathcal{S}(\mathbb{D})$ denotes the set of analytic self maps of \mathbb{D} . Let $\Psi = (\psi_j)_{j=0}^k$ be such that $\psi_j \in \mathcal{H}(\mathbb{D})$ and $\varphi \in \mathcal{S}(\mathbb{D})$. We characterize the boundedness and completely continuous of the sum of generalized weighted composition operators

$$T_{\Psi, \varphi}^k f = \sum_{j=0}^k \psi_j \cdot f^{(j)} \circ \varphi = \sum_{j=0}^k \mathcal{D}_{\psi_j, \varphi}^j f, \quad f \in \mathcal{H}(\mathbb{D}),$$

between weighted Banach spaces of analytic functions $H_v^\infty(H_v^0)$ and $H_w^\infty(H_w^0)$ which unifies the study of products of composition operators, multiplication operators and differentiation operators. As applications, we obtain the boundedness of the generalized weighted composition operators $\mathcal{D}_{\psi, \varphi}^n : \mathcal{B}_v(\mathcal{B}_v^0) \rightarrow \mathcal{B}_w(\mathcal{B}_w^0)$, $\mathcal{B}_v(\mathcal{B}_v^0) \rightarrow H_w^\infty(H_w^0)$ and $H_v^\infty(H_v^0) \rightarrow \mathcal{B}_w(\mathcal{B}_w^0)$, where $\mathcal{B}_v(\mathcal{B}_v^0)$ and $\mathcal{B}_w(\mathcal{B}_w^0)$ are weighted Bloch-type (little Bloch-type) spaces. Also, new characterizations of the boundedness of the operators $T_{\Psi, \varphi}^k$ and $\mathcal{D}_{\psi, \varphi}^n$ are given. Examples of bounded and unbounded operators $T_{\Psi, \varphi}^k$ and $\mathcal{D}_{\psi, \varphi}^n$ are given to explain the role of inducing functions ψ_j , φ and the weights v , w of the underlying weighted spaces.

MSC 2010: 47B33, 47B38, 46E15

Keywords: Composition operators, multiplication operators, generalized weighted composition operators, weighted Banach spaces, weighted Bloch-type spaces, bounded operators

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On the behavior of solutions of Riemann-Liouville fractional nonlinear equation with multiple variable delays

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This paper deals with a nonlinear system of fractional order differential equations with Riemann-Liouville fractional derivative. We prove a new result on the asymptotic stability of zero solution of the considered system. The main result of this paper includes sufficient conditions. The method of the proof is based on the definition of a new Lyapunov-Krasovskiĭ functional. A comparison between our result and the related results in the literature shows that the result of this paper have a general form and extends some former ones. We also present a new example to provide the satisfaction of the conditions of the main result.

Subject Classification: 26A33, 34K20, 34K37, 45J05, 45M10

Keywords: Lyapunov-Krasovskiĭ functional, Riemann-Liouville derivative, nonlinear fractional differential equation, multiple delays, asymptotic stability

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A superconvergent collocation method for two-dimensional Hammerstein integral equations

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We investigate a collocation method for the approximate solution of Hammerstein integral equations in two dimensions. As in [2], collocation is applied to a reformulation of the equation in a new unknown, thus reducing the computational cost and simplifying the implementation. We start with a special type of piecewise linear interpolation over triangles for a reformulation of the equation. This leads to a numerical integration scheme that can then be extended to any bounded domain in \mathbb{R}^2 , which is used in collocation. We analyze and prove the convergence of the method and give error estimates. As the quadrature formula has a higher degree of precision than expected with linear interpolation, the resulting collocation method is superconvergent, meaning that at the collocation nodes it converges faster than over the entire domain, thus requiring fewer iterations for a desired accuracy. We show the applicability of the proposed scheme on several numerical examples and discuss future research ideas in this area.

MSC 2010: 45B05, 65R20, 65D07, 41A15, 47G10

Keywords: Hammerstein integral equations, spline collocation, interpolation, superconvergence

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Approximate solution of the transmission problem via transform method

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There are various approximation methods for solving a system of differential equations, e.g., Adomian decomposition method (ADM), Galerkin method, homotopy perturbation method (HPM), variational iteration method (VIM), and the differential transform method (DTM). The DTM is one of the numerical methods which enables to find an approximate solution in case of linear and nonlinear systems of differential equations. The concept of the differential transform method was first proposed by Zhou [1], who solved linear and nonlinear initial value problems in electric circuit analysis.

In this study, we shall adapt the Differential transform method (DTM) to new type boundary value problem consisting of the equation

$$y''(x) - 4xy'(x) + (4x^2 - 2)y(x) = 0, \quad x \in [0, 1) \cup (1, 2]$$

subject to the boundary conditions given by

$$y(0) = 2, \quad y(2) = 8e^4$$

and additional transmission conditions at the point of interaction $x = 1$, given by

$$y(1+0) = 2y(1-0), \quad y'(1+0) = 3y'(1-0).$$

MSC 2010: 34B05, 65D20

Keywords: Boundary value problems, differential transform method, singular point, transmission conditions

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Hopf bifurcations in a model of epidemic awareness

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This study analyzes a *SIR* based behavioral epidemics model with implicit classes of risky and safe acting susceptibles who update their behaviour according to the prevalence of the disease with a delay. We show that limit cycles via Hopf and Hopf-Hopf bifurcations may emerge due to the coupled delayed feedback effect between behavioral change and epidemic prevalence. We also numerically characterized how the Hopf boundary evolves with respect to the incidence rate of the risky susceptibles, responsiveness of the susceptibles to the observed prevalence of the disease, and the delay term.

MSC 2010: 37G15, 34K18, 34K60

Keywords: SIR model, coupled behavioral-epidemic dynamics, Hopf bifurcation

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Injective modules are ss-lifting in $\mathbf{R}\text{-mod}$

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In this study, we define ss-Harada modules for classifying in rings R for which every injective R -module is ss-lifting. Interaction between ss-Harada modules and ss-lifting modules resolved in the category $\mathbf{R}\text{-mod}$. Thus we have classified ss-lifting modules with the help of injective modules. Firstly we give some properties of ss-Harada modules and rings. Then we characterize ss-Harada modules.

MSC 2010: 16D10, 16D40, 16D60

Keywords: Injective module, lifting module, Harada module, ss-lifting module, ss-Harada module

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A new result on the asymptotic stability of a stochastic delay differential equation of the second order

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This article considers a non-linear stochastic differential equation of second order with multiple constant delays. A new result on the asymptotic stability of the zero solution of that equation is obtained. The new asymptotic stability result is proved by using an appropriate Lyapunov-Krasovskii functional. As a numerical application, an example is constructed to verify the assumptions of the stability result. The given result extends and improves some former results in the literature.

Subject Classification: 34K20, 34K50, 60H35

Keywords: Stochastic differential equation, second order, stability, Ito formula, Lyapunov-Krasovskii functional

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Topological group-groupoids

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The groupoid was offered by Brandt[1]. The structure of topological groupoid was given by Ehresmann [2]. The groupoid action is very important appliance in algebraic topology which is offered by Ehresmann. Another algebraic notion is a covering which is given by Brown [3]. The topological group-groupoids (G -groupoid) was first offered by A.F.Ozcan [4]. The definition of coverings of topological G -groupoid and actions of topological G -groupoid were also given by A.F.Ozcan [5]. In this paper, we will create a category $TGGpdCov(G)$ of coverings of topological G -groupoid and a category $TGGpdOp(G)$ of actions of topological G -groupoid. And then we will prove that this categories are equivalent.

MSC 2010: 22A22, 14H30, 57M60

Keywords: Groupoids, coverings of groupoid, actions of groupoid

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The minus partial order on endomorphism rings

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Let M be right module and $S := \text{End}(M)$ be the ring of endomorphism of M . The minus partial order was first extensively studied [1] and [2]. The authors introduced in [3] the minus partial order on a ring. In this talk, we define the minus partial order of S and give some generalization of minus partial orders.

MSC 2010: 316D80; 16D40; 16W20

Keywords: Endomorphism ring, minus partial order operator, Rickart ring

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Topological group and small loop transfer space

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In [1], Brodskiy et al. proved that if the operation of taking inverse in $\pi_1^{wh}(X, x_0)$ is continuous, then $\pi_1^{wh}(X, x_0)$ is a topological group. In [2], Pashaei et al. showed that if X is a small loop transfer space at x_0 , then $\pi_1^{wh}(X, x_0)$ is a topological group. In this talk, we show that if X is a $\pi_1^{gc}(X, x_0)$ -small loop transfer space at x_0 , then $\pi_1^{wh}(X, x_0)$ is a topological group.

MSC 2010: 54H11, 57M05, 57M07

Keywords: Topological groups, fundamental group, Whisker topology

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Existence and nonexistence for the higher-order logarithmic Klein-Gordon equation

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In this presentation, we consider a Klein-Gordon equation with logarithmic nonlinearity. The Klein-Gordon equation is a kind of evolution equation. The evolution equations, namely partial differential equations with time t as one of the independent variables. Firstly, we established the global existence of solution by potential well method. In addition, we also obtain results of decay and nonexistence of solutions.

MSC 2010: 35A01, 35B44, 35G20

Keywords: Klein-Gordon, global existence, nonexistence

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Existence and decay for the logarithmic Lamé system

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In this presentation, we consider the logarithmic Lamé system. The logarithmic Lamé system is a kind of evolution equation. The evolution equations, namely partial differential equations with time t as one of the independent variables. Firstly, we established the existence of solution by semigroup method. In addition, we proved the decay of solutions.

MSC 2010: 35B40, 35L05, 35L70

Keywords: Decay, existence, Lamé system

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Multipliers of frames and woven frames in Hilbert spaces

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The concept of frames, as a generalization of the bases in Hilbert spaces, were first introduced by Duffin and Schaeffer [3] during their study of nonharmonic Fourier series in 1952. In 1985, as the wavelet era began, Daubechies, Grossman and Meyer [2] observed that frames can be used to find series expansions $L^2(\mathbb{R})$ which are very similar to the expansions using orthonormal bases.

Now frame theory has been widely used in many fields such as filter bank theory, image processing, particularly in the more specialized context of wavelet frames and Gabor frames. Multipliers are operators which have important applications for signal processing and acoustics [5, 4]. Also woven and weaving Bessel sequences and frames is a very important and practical tools in the applications of frames [1]. In this study, we define the notion of multiplier for woven and weaving frames and we show that the properties of multiplier continuously depends on the chosen symbol sequence m and chosen two woven Bessel sequences. Further, we study the stability of woven frames under perturbation and its connection with multipliers.

MSC 2010: 42C15, 42C40, 47A05

Keywords: Frame, multiplier, woven frame, Bessel sequence

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On controlled frame multipliers in Hilbert spaces

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Frames for Hilbert spaces were first introduced by Duffin and Schaeffer [2] in 1952 in order to investigate some deep problems in non-harmonic Fourier series. They were reintroduced in 1986 by Daubechies, Grossman, and Meyer and popularized from then on. The redundancy and flexibility offered by frames have provided a setting for their application in several areas of mathematics, physics and engineering. Owing to the increasing number of applications, considerable attention and effort have been paid towards development of frame theory in recent years. For basic results on frames, see [1].

A frame is a countable family of elements in a separable Hilbert space, which allows stable but not necessarily unique decomposition of arbitrary elements into expansion of the frame elements. Given a separable Hilbert space \mathcal{H} with inner product $\langle \cdot, \cdot \rangle$, a sequence $\Psi = \{\psi_j\}_{j \in J}$ is called a *frame* for \mathcal{H} if there exist constants $0 < A \leq B < \infty$ such that

$$A\|f\|^2 \leq \sum_{j \in J} |\langle f, \psi_j \rangle|^2 \leq B\|f\|^2, \quad \text{for all } f \in \mathcal{H}. \quad (1.1)$$

If the upper bound holds in the above inequality, then $\{\psi_j\}_{j \in J}$ is said to be a *Bessel sequence* with Bessel constant B . A tight frame refers to the case when $A = B$, and a *Parseval frame* refers to the case when $A = B = 1$.

Here, we study multipliers for controlled frames and investigate some of their properties. Furthermore, by using controlled dual frame, we show that the inverse of a multiplier operator is a multiplier operator too. For more details, see [3].

MSC 2010: 42C15, 42C40, 41A58

Keywords: Frame, controlled frame, multiplier

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Woven continuous g -fusion frames in Hilbert spaces

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Frames for a Hilbert space were first introduced by Duffin and Schaeffer [2] in 1952. Daubechies, Grossmann and Meyer reintroduced frames, in 1986 and considered from then. Frame theory has applications in signal processing, image processing, data compression and sampling theory.

Some new types and generalizations of frame were introduced by researchers such as fusion frames, g -frames, woven frames, continuous frames

In other side, weaving frames were introduced by Bemrose et.al. [1] in 2016. Weaving frames are powerful tools for pre-processing signals and distributed data processing. Many researchers studied and generalized weaving frames. Some of these generalizations are weaving g -frames, weaving g -fusion frames [3].

In this paper, motivated and inspired by the above-mentioned works we introduce the concept of weaving continuous g -fusion frame. This frame includes weaving g -frames and weaving g -fusion frames. We extend some of the recent results of standard woven frames and woven fusion frames to woven continuous g -fusion frames.

MSC 2010: 42C15, 42C30, 42C40

Keywords: Frame, continuous g -fusion frame, weaving

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Cubic b-spline collocation technique for time-dependent problems with small perturbation parameter

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In this talk, an efficient numerical method is successfully applied to treat a time-dependent singularly perturbed problem whose solution exhibits boundary layers. The key to the proposed method is splitting the solution into a smooth and a singular component, which is reconstructed using known additional information about the location of a layer. We obtain an approximate solution by using a Galerkin finite element with cubic b-spline functions as element shape and weight functions. The resulting system is fully discretized using the Crank Nicolson time discretization scheme. Test problems demonstrate the accuracy and efficacy of the presented method. The numerical results are found to be in good agreement with the exact solutions.

MSC 2010: 65L11, 65M60, 65D07

Keywords: Singularly perturbed problems, asymptotic expansion, regular component, singular component, Galerkin finite element method, cubic b-spline, Crank Nicolson scheme

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A new method for variable order fractional Volterra-Fredholm integro-differential equations

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In this article, a new method is developed for nonlinear variable order fractional Volterra-Fredholm integro-differential equations. The fractional derivatives are taken in the Caputo sense. This approach is based on reproducing kernel which is constructed by shifted Legendre polynomials. In order to show the robustness of the proposed method, some examples are solved and numerical results are given in tabulated forms.

MSC 2010: 46E22, 47B32, 26A33

Keywords: Reproducing kernel method, Legendre polynomials, Caputo derivative, variable order, Volterra-Fredholm integro-differential equations

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New oscillation criteria for odd-order neutral differential equations with distributed deviating arguments

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New sufficient conditions for the oscillation of all solutions to a class of odd-order differential equations with unbounded neutral coefficients and distributed deviating arguments are established. The results are obtained by a Riccati type transformation as well as by an integral criterion. Examples illustrating the results are provided and some suggestions for further research are indicated.

MSC 2010: 34C10, 34K11, 34K40

Keywords: Oscillation, asymptotic behavior, odd-order, neutral differential equation

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Conjectures of Rossi and Sally on the monotonicity of the Hilbert functions

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Characterizing numerical functions that might be Hilbert functions of one dimensional Cohen-Macaulay local rings is an important problem of commutative algebra. Sally's conjecture says that

"The Hilbert function of a one dimensional Cohen-Macaulay local ring with small enough embedding dimension is nondecreasing."

and Rossi's conjecture says that

"The Hilbert function of a one dimensional Gorenstein local ring is non-decreasing. "

In this talk, we will talk about the history and the recent results on these conjectures and the monotonicity of the Hilbert Functions.

MSC 2010: 13H10, 14H20, 13P10

Keywords: Pseudosymmetric semigroups, symmetric semigroups, Cohen-Macaulay local rings, Hilbert functions

Some Korovkin theorems for linear operators via power series method

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The power series methods, include both Abel and Borel methods, are well known and more effective than ordinary convergence. These methods are considered in the Korovkin type approximation theory first time with the Abel summability method in [5] and in many papers the authors show how this concept can be applied to approximation theory [3, 4, 7]. Recently, by relaxing the positivity condition on linear operators, various approximation theorems have also been obtained. For instance, Duman and Anastassiou [1] relaxed the positivity condition of linear operators in the Korovkin-type approximation theory via the concept of statistical convergence that is another interesting convergence method [2, 6] and as it is well known that power series methods and statistical convergence are incompatible.

In the present work the main aim is to study some Korovkin-type approximation theorems for linear operators defined on derivatives of functions via power series methods. We give an example that our theorem makes more sense. It should also be noted that we study the rate of convergence. In the final section we summarize the results obtained in present paper.

MSC 2010: 40A35, 40C05, 40G10

Keywords: Korovkin type approximation theory, power series method, rate of convergence, linear operators

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On ωe^* -continuous functions

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One of the most important notions of topology is undoubtedly the notion of open set. Many weak forms of the notion of the open set such as e^* -open sets [5], β -open sets [1], ω -open sets [4], $\omega\beta$ -open sets [2] have been studied by several authors. Recently, many fundamental results related to the notions mentioned above have been obtained. In addition, the notion of continuity is one of the most important notions as well as the notion of open sets. Several weak and strong forms of the notion of continuity such as ω -continuity [6], e^* -continuity [5] and $\omega\beta$ -continuity [5] have been studied by many mathematicians and they have been obtained many characterizations and fundamental properties. In this study, we introduce and investigate the notion of ωe^* -continuous function via ωe^* -open set [7]. We have obtained some characterizations and some fundamental properties of them. Also, the relationships between this notion and the other types of continuity in the literature are revealed. Furthermore, some fundamental results related to subspaces, restriction functions, product spaces, and graph functions have been obtained.

MSC 2010: 54C05, 54C08, 54C10

Keywords: ωe^* -open set, ωe^* -neighborhood, ωe^* -continuity

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On the Hyers-Ulam stability of nonlinear Volterra integro-differential equation with the multiple constant delays

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This paper deals with a nonlinear Volterra integro-differential equation with the multiple constant delays. We prove two theorems on the Hyers-Ulam and Hyers-Ulam-Rassias stabilities of the equation on a finite interval. The given theorems include sufficient conditions and the technique of the proofs is based on the fixed point method and Picard operator theory, and Pachpatte's inequality. Our results allow new contributions to the topic of Hyers-Ulam and Hyers-Ulam-Rassias. We give an example to provide and illustrate the application of our results.

MSC 2010: 34K05, 34K30, 47G20

Keywords: Hyers-Ulam stability, Hyers-Ulam-Rassias, delay Volterra integro-differential equation, fixed point, Picard operator

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A block conjugate gradient method for linear positive definite quaternion matrix equations

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A quaternion linear matrix equation can be transformed into a real linear matrix equation. During this transformation into the real setting, the dimension of the matrix equation increases by four times. Applications of Krylov subspace based approaches to these problems whose sizes are four times those of the original problems require more storage, but then we can benefit from fast and stable algorithms operating on real arithmetic. This study aims at the solution of a linear quaternion matrix equation with a Hermitian and positive definite coefficient matrix by employing the conjugate gradient method. We consider the setting when the quaternion Hermitian positive definite matrix at hand is very large so that direct methods such as those based on a Cholesky factorization are not applicable. First, we transform the quaternion linear matrix equation into a real matrix equation. Then a block conjugate gradient method is applied to the real matrix equation. The solution obtained after applying the conjugate gradient method is the real representation of the solution of the original quaternion problem. Thus, a conversion of this real solution to the quaternion setting is performed in the end.

MSC 2010: 65F10, 65F20, 15A24, 15A06, 15B33

Keywords: Krylov subspace, conjugate gradient, quaternion matrix equation, skew-field

On Maki's Λ -sets via strong β - I -open set

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In this presentation, we will define the notions of strong β - I -kernel of a set, $\Lambda_{s\beta I}$ -sets, generalized $\Lambda_{s\beta I}$ -sets, $\lambda_{s\beta I}$ -closed sets by using the concepts of strong β - I -open sets in ideal topological spaces. It is well known that the notion of strong β - I -open sets is given by Hatir, Keskin and Noiri [10] and this notion is weaker than the notion of pre- I -open sets. Several characteristics will be studied. Also, two low separation axioms, namely strong β - I - T_1 spaces and strong β - I - T_2 spaces will be presented.

MSC 2010: 54A10, 54D10, 54A99

Keywords: Strong β - I -open sets, strong β - I -kernel, $\Lambda_{s\beta I}$ -sets, generalized $\Lambda_{s\beta I}$ -sets, $\lambda_{s\beta I}$ -closed sets

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Weighted composition operators on weighted spaces of holomorphic functions on Banach spaces

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The purpose of this presentation is to investigate qualitative properties of weighted composition operators between weighted spaces of holomorphic functions defined on the open unit ball of a Banach space. Necessary and sufficient conditions are given for weighted composition operators to be compact. We refer to [1], [2], [3], and [4] for more details about weighted composition operators.

MSC 2010: 47B38, 46G20

Keywords: Weights, weighted spaces, weighted composition operators

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Benchmarking for nonsmooth convex optimization methods

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The aim of this study is to provide a brief overview of nonsmooth convex optimization methods by discussing their implementation and comparing them using some numerical results. The test problems used in the numerical experience include small and large-scale nonsmooth convex and nonconvex optimization problems with special structures.

This study will be concluded by providing a brief discussion on future directions and applications of nonsmooth optimization.

MSC 2010: 65K05, 90C25, 90C06

Keywords: Nonsmooth optimization, convex optimization, comparison, large-scale problems

A note on qualitative behaviors of solutions of integro-differential equations

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In this paper, we study some properties of solutions to an integro-differential equation. A few new results have been given on the fundamental properties of solutions of the considered equation. The Lyapunov-Krasovkii functional approach is utilized to prove the main findings of this study. An example is provided to demonstrate how the results can be applied.

MSC 2010: 34D05, 34K20, 45J05

Keywords: Boundedness, stability, instability, integrability

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Enhanced new qualitative criteria to delay fractional integro-differential equations with Caputo derivative

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The aim of this article is to obtain some new and enhanced qualitative criteria to solutions of certain nonlinear delay fractional integro-differential equations with Caputo fractional derivative. The enhanced qualitative criteria are related to uniform stability, asymptotic stability, Mittag-Leffler stability and boundedness of solutions of the delay fractional integro-differential equations with Caputo derivative. The new results of this paper are proved by defining a suitable Lyapunov function and applying the Lyapunov–Razumikhin method. Here we improved some results that are available in the literature and obtain that results under less restrictive conditions. Two examples are also provided to illustrate the obtained results.

Subject Classification: 34K20, 34K50, 60H35

Keywords: Fractional order, delay, stability, Lyapunov-Krasovskii functional

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Wsa-coinjective modules

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Throughout this talk, all rings considered are associative with an identity element and all modules at hand are right and unital.

It is shown that over an arbitrary ring the class of all short exact sequences $0 \longrightarrow M \xrightarrow{\psi} N \xrightarrow{\phi} K \longrightarrow 0$ such that $\text{Im}(\psi)$ is a wsa-supplement in N is a proper class. We study the objects of this class, which we call \mathcal{WSS} . We show that a module M is \mathcal{WSS} -coinjective if and only if it is a wsa-supplement $E(M)$. We prove that over a right CC -ring a projective module P is \mathcal{WSS} -coinjective if and only if $P/\text{wsa}(P)$ is injective. We also prove that a ring R is weakly semiartinian if and only if every right R -module is \mathcal{WSS} -coinjective. Finally, we show that over a crumbling-free ring \mathcal{WSS} -coprojective modules are only the projective modules.

MSC 2010:16D10, 18G25

Keywords: wsa-supplement submodule, weakly semiartinian module, wsa-coinjective module

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Some new sequence spaces derived by the composition of weighted mean and quadruple band matrix

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In this study, some new sequence spaces, $c_0^{u,v}(\tilde{F})$, $c^{u,v}(\tilde{F})$, $l_\infty^{u,v}(\tilde{F})$ and $l_p^{u,v}(\tilde{F})$ ($1 \leq p < \infty$) derived by the composition of the generalized weighted mean matrix $F(u, v)$ and the quadruple band matrix $Q(r, s, t, z)$, are defined and it was shown that these spaces are linearly isomorphic to c_0, c, l_∞ and l_p ($1 \leq p < \infty$) spaces. Furthermore, we give the Schauder basis of the spaces $c_0^{u,v}(\tilde{F})$, $c^{u,v}(\tilde{F})$ and $l_p^{u,v}(\tilde{F})$ ($1 \leq p < \infty$). After then, we give some inclusion relations related to these spaces and determine α -, β - and γ -duals. Finally, we characterize some matrix classes related to these spaces.

MSC 2010: 40C05, 40H05, 46B45

Keywords: Matrix domain, Schauder basis, α -, β - and γ -duals, matrix classes

Complex valued controlled fuzzy metric spaces and some common fixed point results

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In this paper, we introduce a new extension of fuzzy b -metric space, namely complex valued controlled fuzzy metric space. Our notion is the generalization of fuzzy b -metric space in terms of complex valued fuzzy metric space. We also prove Banach type contraction principle in this new version of complex valued fuzzy metric space. A fixed point theorem satisfying a new generalized ϕ -contraction condition is established. Furthermore, we furnish an example to validate our result.

MSC 2010: 47H10, 54H25, 37C25

Keywords: Complex fuzzy set, complex valued fuzzy metric space, controlled fuzzy metric space

On bicomplex numbers with coefficients from higher order Fibonacci numbers

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In this study, bicomplex higher order Fibonacci numbers that generalize the bicomplex Fibonacci numbers are studied. Several identities such as Binet's formula, recurrence relation, ordinary generating function, exponential generating function, Poisson generating function for bicomplex higher order Fibonacci numbers are given. Furthermore, Vajda's identity, Catalan's identity, Cassini's identity and d'Ocagne's identity involving these numbers are derived.

MSC 2010: 11B37, 11B39, 20G20, 11R52

Keywords: Fibonacci number, higher order Fibonacci number, bicomplex number

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New continuity types via local closure function

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In this study, we introduce the concepts of Γ - \mathcal{I} -open set and pre- Γ - \mathcal{I} -open by using local function in ideal topological spaces. The relationship between these new types of subsets and some previously defined subset types is showed. Moreover, we define new continuity types with the help of classes of Γ - \mathcal{I} -open set and pre- Γ - \mathcal{I} -open. Thanks to these new types, we obtain the decomposition of continuity and Γ - \mathcal{I} -continuity.

MSC 2010: 54A05, 54A10, 54C08, 54C10

Keywords: Ideal topological spaces, local function, local closure function, continuity, decomposition of continuity

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A new characterization of reflexive rings and their applications

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Let R be a ring and $Z(R)$ the center of R . The ring R is called central right idempotent reflexive if $aRe = 0$ implies $eRa \subseteq Z(R)$ for any $a \in R$, $e^2 = e \in R$. We investigate properties of this class of rings. We obtain additionally that the semiprime rings, central semicommutative rings and central reversible rings are central right idempotent reflexive. It is shown that both the polynomial rings and the power series ring over a central right idempotent reflexive ring are central right idempotent reflexive.

MSC 2010: 16N40, 16S70, 16S80, 16U20

Keywords: Reflexive ring, central right idempotent reflexive, central left idempotent reflexive, central semicommutative ring, central reversible ring

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On statistical convergence for triple sequences on L -fuzzy normed space

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On L -fuzzy normed spaces, which is a generalization of normed spaces, statistical convergence, statistical Cauchy, statistical bounded concepts for both classic and double sequences have been defined recently and the relationships between these concepts have been examined. [9] In this study, the concepts of statistical convergence, statistical Cauchy and statistical boundedness on triple sequences will be given and the relationships between them will be examined with respect to L -fuzzy norm ρ .

MSC 2010: 03E72, 40A05, 40E05, 40G05

Keywords: Triple sequences, statistical convergence, statistical Cauchy, statistical boundedness, L -fuzzy normed space

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Quantum computer-resistant digital signature generation using complex numbers

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Using complex numbers in digital signature processes is a method with increased security that can be used both in quantum computers and classical computers [1]. The American National Security Agency Central Security Service [2] continues its studies on this subject in the form of competitions. Digital signature technologies concern all segments of society and have high economical added value. It is a technology that converts a numerical value you produce into money. It is used for secure data transmission, especially for communication between computers. Especially with the spread of quantum computers, it is seen with research that there will be a significant gap in cryptology [3, 4]. The technique considered within the scope of this study is the realisation of Quantum Digital Signature Generation Using Complex Numbers.

Based on Complex Numbers Digital signature consists of a set of parameters used to define the signature and integrity of the message, as in the classical signature. A signer's private key V_a , such as Alice, is used to sign a message, and the corresponding public key is used for signature verification. A random number k is used to generate the signature for each message. $(k, n - 1) = 1$, and the greatest common divisor (GCD) of k must be chosen secretly by the signer. The required signature and signature lengths are $S = (r, s)$ and $|\text{hash}(M)| + |q|$ [1], respectively.

MSC 2020: 68Q09, 68Q12, 81-04

Keywords: Complex number, quantum computing, digital signature, quantum digital signature

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Analysis of global exponential stability and pseudo almost periodic solution of a class of chaotic neural networks on time scales

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In this paper, by using the exponential dichotomy of linear dynamic equations on time scales, a fixed point theorem and the theory of calculus on time scales, we obtain sufficient conditions for the existence, uniqueness and global exponential stability of weighted pseudo-almost periodic solutions of a class of chaotic neural networks with mixed delays on time scales. A numerical example is also presented to illustrate the feasibility of our results.

MSC 2010: 34K14, 92B20, 93D20

Keywords: Weighted pseudo-almost periodic solutions, chaotic neural networks, time scales, mixed delays, existence and uniqueness, exponential stability

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An exact penalty function approach for inequality constrained optimization problems based on a new smoothing technique

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Exact penalty methods are one of the effective tools to solve nonlinear programming problems with inequality constraints. In this study, a new family of smoothing technique to lower order exact penalty functions is introduced. Error estimations are presented among the original, non-smooth exact penalty and smoothed exact penalty problems. It is proved that an optimal solution of smoothed penalty problem is an optimal solution of original problem. A smoothing penalty algorithm based on the the new smoothing technique is proposed and the convergence of the algorithm is discussed. Finally, the efficiency of the algorithm on some numerical examples is illustrated.

MSC 2010: 90C30, 65K05, 65D15

Keywords: Constrained optimization, smoothing technique, exact penalty function

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Roman type domination numbers of Fibonacci cubes

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The Fibonacci cube of dimension n is the subgraph of n dimensional hypercube induced by the binary strings of length n that contain no two consecutive ones [1]. These graphs are applicable as interconnection networks and in theoretical chemistry, and lead to the Fibonacci dimension of a graph [2]. The domination problem is a difficult problem to deal with in general. Let G be a graph and the D be a subset of the vertex set $V(G)$ of G . D is a dominating set if every vertex from $V(G)$ either belongs to D or adjacent to some vertex from D . The domination number is the minimum cardinality of a dominating set of G . In this work we consider some special variants of domination problem called Roman type domination problem [3]. We remark that Roman domination problem is applicable both as a war defense strategy and can be applicable for a huge number of recourse-sharing problems. For instance, emergency vehicles are very expensive, and minimizing their numbers, while still being able to help injured people, will be of great use. We consider three Roman Domination type invariants called Roman domination number, double Roman domination number and weak Roman domination number and determine their exact values for Fibonacci cubes of dimension at most 10 by using their structure and Integer Linear Programming.

MSC 2010: 05C69, 68R10, 90C10

Keywords: Fibonacci cube, hypercube, Roman domination number

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Bezier curves belonging to some surfaces

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Bernstein polynomials, which have important applications in many areas and more particularly commonly utilized in the scope of computer programming, are examined and some significant features of Bernstein polynomials are mentioned. Definition and characteristics of a Bezier curve in terms of Bernstein polynomials and De Casteljaou algorithms that contributed to the development of Bezier curves and surface concepts are investigated, as well. A curve on surface, as well as multiple points on this curve, are taken in order to calculate the Bezier curve belonging to these points. The Bezier curves on the surface curves on are investigated and plotted with Matlab of which outputs are also included in the study.

MSC 2010: 65D17, 14H50, 51N05, 68U07

Keywords: Bernstein polynomials, Bezier curves, De Casteljaou algorithms, Matlab

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Improved new results on the asymptotic admissibility of nonlinear singular systems with multiple delays

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In this paper, we study the asymptotic admissibility of a class of nonlinear singular systems of first order with two time-varying delays. We use the Lyapunov- Krasovskii functional method and some well-known inequalities to prove the results. Moreover, we give some examples with numerical simulations to show the correctness of the applied method by MATLAB-Simulink. By this work, we extend and improve some results available in the literature.

MSC 2010: 34A08, 34K20, 34K40

Keywords: Asymptotic admissibility, impulse-free, regular, singular system

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On the geometry of Ricci solitons

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Einstein manifolds have an important place in manifold theory. As a generalization of Einstein manifolds, Ricci solitons have gained considerable importance and they have been studied widely by many mathematicians recently in literature. The purpose of this work is to study Ricci solitons, which are the special solutions of Ricci flows, on Riemannian manifolds. We obtain some significant results related to such solitons.

MSC 2010: 53B20, 53C44, 35Q51

Keywords: Ricci soliton, Riemannian manifold, affine vector field

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Index

- Özdin, Tufan, 126
Özcan, Abdullah Fatih, 104, 125
Özkoç, Murad, 138
Öztürk, Havva Nur, 106
İbrahimoglu, Betül Guduk, 94
İlter, Serkan, 80
İçen, İlhan, 104, 125
İğret Araz, Seda, 105
İlter, Serkan, 106
Çınkır, Zübeyir, 70
Çekiç, Gökçen, 66
Çete, Ayfer, 67
Çetin, Pakize, 68
Çetin, Şahap, 69
Şaşmaz, Pmar, 138
Şahin Bayram, Nilay, 137
Şahin, Nil, 136
Şengül, Mustafa Taylan, 94
Şengün, Merve, 139
Şimşek, Sinem, 140
- Acar, Hülya, 47
Acharjee, Santanu, 138
Agrawal, Garvita, 40
Ait-Amrane, Rosa, 41
Akkus, Ilker, 42, 43
Aksoy, Ruken, 128
Aktaş, Ömer, 107
Aldemir, Mehmet Serif, 44
Ali, Imran, 45
Alsoy-Akgün, Nagehan, 46
Amirali, Ilhame, 47
Arık, İrem, 49
Arab, Zineb, 48
Arar, Nouria, 116, 133
Arslan, Berna, 50
Arslan, Okan, 51
Artıran, Merve, 52
Ata, Enes, 53
Ateş, Muzaffer, 54, 55
Aydemir, Kadriye, 121
Ayid Ali, Rodi, 55
- Başaran, Hamdullah, 99
Badawi, Ayman, 25
Bagirov, Adil, 27
Baydaş, Şenay, 157
Belbachir, Hacene, 41
Biçer, Harun, 56
- Bilgin, Aygül, 80
Bodur, Murat, 57
Bohner, Martin, 28
Bouchal, Lydia, 58
Boutiche, Mohamed Amine, 101
- Cakir, Musa, 93
Can, Zeynep, 59
Cancan, Murat, 44
Cansu, Sibel, 60
Chehrazi, Akram, 61
Ciftci, Idris, 62, 63
Coşkun, Mustafa, 64
Coskun, Nimet, 65
- Dalmanoğlu, Özge, 71
Daraby, Bayaz, 130
Dede, Cahit, 72
Deliktas-Ozdemir, Ekin, 73
Demir, Nihal, 75
Demirci, Yılmaz Mehmet, 74, 123, 146
Demirel, Ceyda, 76
Denizler, İsmail Hakkı, 77
Denizler, İsmail Hakkı, 117
Dinç, Yavuz, 78
Dogru Akgöl, Sibel, 79
Duru, Hülya, 80, 106
- Ediz, Suleyman, 44, 63
Ekmekçi, Ramazan, 81
El-Sayed, Ahmed Mohamed, 29
Erçelik, Elif, 82
Erdoğan, Betül, 42
Erdur, Sultan , 83
Ersu, Sare Buse, 84
Eryılmaz, Figen, 85
Eskisehirli, Beyaz Basak, 86
Evcin, Cansu, 87
- Fındık, Şehmus, 89
Fellahi, Boutheina, 88
- Gül, Kübra, 95
Güleryüz, Rabia, 96
Gümüş, Hafize, 97
Gümüş, Hafize, 75
Güneş Aktaş, Çisem, 98
Gürdal, Mehmet, 99
Gözen, Melek, 91
Genç, Ahmet, 51

- Gorentas, Necat, 127
 Guettai, Ghania, 92
 Gunes, Baransel, 93
 Hamidođlu, Ali, 100
 Hassi, Ilham, 101
 Hedayatian, Karim, 102
 Hioual, Amel, 103, 116
 Iqbal, Javid, 45
 Jung, Soon-Mo, 30
 Köse, Handan, 151
 Küsmüş, Ömer, 115
 Kılıç, Nurten, 112
 Kızılaslan, Gonca, 113
 Kara, Merve, 107
 Karakaş, Bülent, 157
 Kartal, Neriman, 108
 Kaya Gök, Gülistan, 109
 Kayar, Zeynep, 110, 111
 Kaymakçalan, Billur, 110, 111
 Keskin Kaymakçı, Aynur, 141, 150
 Khalili Golmankhaneh, Alireza, 31
 Koca, Seyit, 106
 Korkmaz, Erdal, 152
 Kutlu, Mustafa Kutay, 114
 Kuzu, Okan, 68
 López-Alfonso, Salvador, 32
 López-Pellicer, Manuel, 32
 Laouar, Zineb, 103, 116
 Luzum, Murat, 117
 Manhas, Jasbir Singh, 118
 Mansız, Kasım, 119
 Mebarki, Karima, 58
 Merikhi, Bachir, 88
 Mermut, Engin, 114
 Micula, Sanda, 120
 Mohammadrezaee, Maryam, 132
 Moll-López, Santiago, 32
 Mukhtarov, Oktay Sh., 121
 Musali, Ali, 122
 Nadar, Mustafa, 82
 Namegoshayfard, Mohammad, 102
 Nazari, Akbar, 132
 Nişancı Türkmen, Burcu, 123
 Ogut, Hatice, 155
 Oktan, Zozan, 124
 Ouannas, Adel, 103
 Oussaeif, Taki Eddine, 103
 Paolucci, Alessandro, 33
 Park, Choonkil, 34
 Pashaei, Seyed Zeynal, 127
 Patel, Prashantkumar, 57
 Peker Dobie, Ayse, 73
 Pişkin, Erhan, 78, 128, 129
 Pignotti, Cristina, 33
 Pinelas, Sandra, 35
 Prajapati, Gendlal, 148
 Rahimi, Asghar, 130
 Rahmani, Mourad, 92
 Rajeswari, K. N., 40
 Rashidi-Kouchi, Mehdi, 131, 132
 Redouane, Kelthoum Lina, 133
 Sönmez, Abdulcabbar, 147
 Sağlam, Çağrı, 122
 Sakar, Mehmet Giyas, 134
 Sancar, Erkan, 129
 Sarıgül, Mine, 135
 Saygı, Zülfükar, 52, 156
 Schröcker, Hans-Peter, 36
 Sharma, Rajesh Kumar, 148
 Slynko, Vitalii, 37
 Türkmen, Ergül, 146
 Türkođlu, Cennet, 147
 Tan, Elif, 41
 Teeth, Manjeet Singh, 40
 Tekin, Şerife, 141
 Topal, Hayri, 142
 Tor, Ali Hakan, 143
 Tunç, Cemil, 48, 49, 78, 83, 119, 124, 139, 145
 Tunç, Ercan, 135
 Tunç, Osman, 144, 145
 Ustaogđlu, Uğur, 60
 Uyar, Burak, 96
 Yüksel, Kerem, 122
 Yılmaz Luzum, Ceyda, 157
 Yılmaz, Erol, 60
 Yılmaz, Melis Berçin, 156
 Yılmaz, Nurullah, 155
 Yılmaz, Yalçın, 69
 Yaşar, Ahmet Buğra, 153
 Yaşar, Cumali, 153
 Yağmur, Tülay, 149
 Yadav, Gopal, 148
 Yakar, Coşkun, 69

Yalçın, Ruhi, 151
Yalaz, Ferit, 150
Yapalı, Reha, 152
Yazgan, Ramazan, 154
Yazlık, Yasin, 67

Yeniaydın, Safak, 43
Yiğit, Abdullah, 158
Yoldaş, Halil İbrahim, 159
Yucel, Merve, 121